

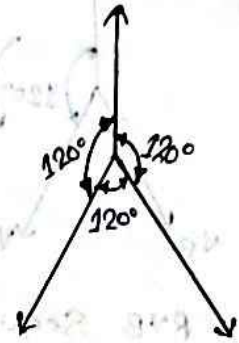
# EEE UNIT-1 Phasor Representation of 3-Phase Voltages

The each phase voltage is given by

$$V_R = V_m \sin \omega t$$

$$V_Y = V_m \sin (\omega t - 120^\circ)$$

$$V_B = V_m \sin (\omega t - 240^\circ)$$



At any instant, the algebraic sum of voltages is equal to zero

$$V_R + V_Y + V_B = 0$$

$$= V_m \sin \omega t + V_m \sin (\omega t - 120^\circ) + V_m \sin (\omega t - 240^\circ)$$

$$= V_m \sin \omega t + V_m [\sin \omega t \cos 120^\circ - \cos \omega t \sin 120^\circ] +$$

$$V_m [\sin \omega t \cos 240^\circ - \cos \omega t \sin 240^\circ]$$

$$= V_m \sin \omega t + V_m [\sin \omega t \cos (90^\circ + 30^\circ) - \cos \omega t \sin (90^\circ + 30^\circ) +$$

$$V_m \sin \omega t \cos (180^\circ + 60^\circ) - \cos \omega t \sin (180^\circ + 60^\circ)]$$

$$= V_m \sin \omega t + V_m [\sin \omega t (\frac{1}{2}) - \cos \omega t (\frac{\sqrt{3}}{2})] + V_m [\sin \omega t (\frac{1}{2})$$

$$- \cos \omega t (-\frac{\sqrt{3}}{2})]$$

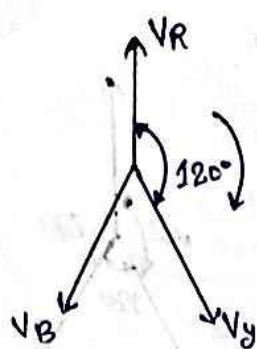
$$= V_m \sin \omega t + \cancel{V_m \sin \omega t (\frac{1}{2})} - \cancel{\cos \omega t (\frac{\sqrt{3}}{2})} - \cancel{V_m \sin \omega t (\frac{1}{2})} + \cos \omega t (\frac{\sqrt{3}}{2})$$

$$V_m \sin \omega t = 0$$

Phase Sequence :- The Sequence of attaining maximum values by three phase voltages (or) current is known as phase Sequence.

- (i) Positive phase Sequence
- (ii) Negative phase Sequence

Positive phase sequence = In this phase sequence the field is



rotated in clock wise direction & This phase sequence is also called as RYB phase sequence

& The voltage in each phase is given by.

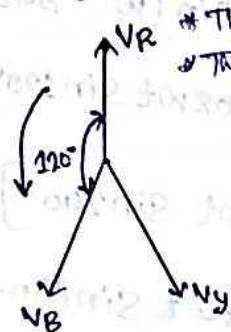
$$V_R = V_m \sin \omega t$$

$$V_Y = V_m \sin (\omega t - 120^\circ)$$

$$V_B = V_m \sin (\omega t - 240^\circ)$$

RYB Sequence

Negative phase sequence = This phase sequence is rotated in



anti-clockwise direction. & This phase sequence is also called as RBY phase sequence & The voltage in each phase is given by,

$$V_R = V_m \sin \omega t$$

$$V_B = V_m \sin (\omega t - 120^\circ)$$

$$V_Y = V_m \sin (\omega t - 240^\circ)$$

RBY Sequence

Interconnection of 3-phase system!

(i) Star Connection

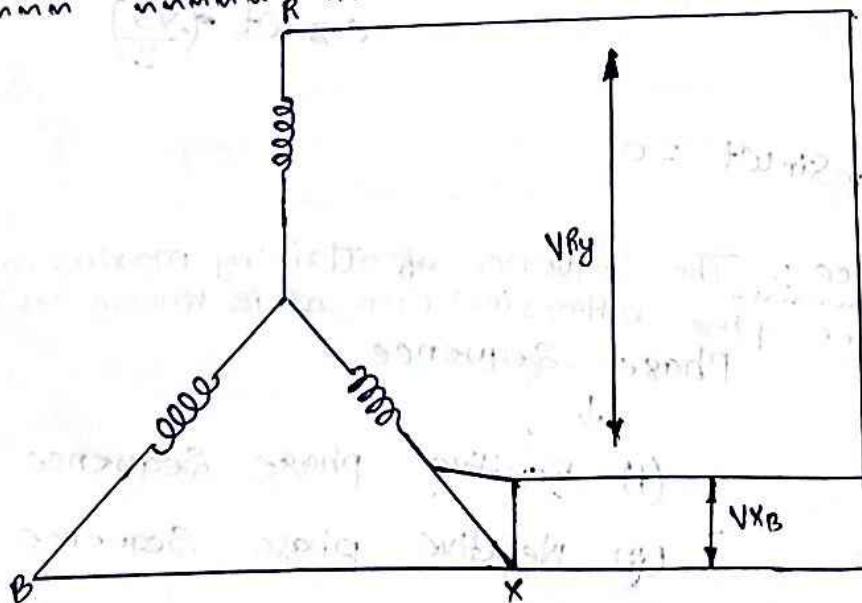
(ii) Delta Connection



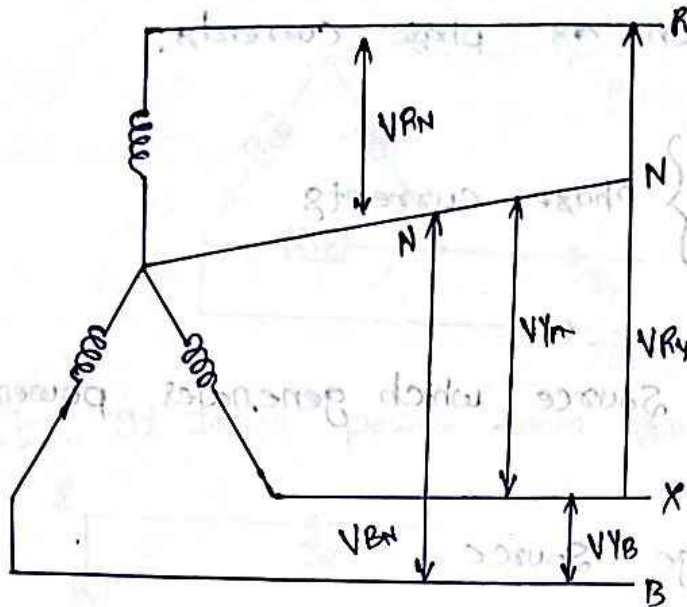
3-φ, 3-wire connection ;

3-φ 4-wire connection.

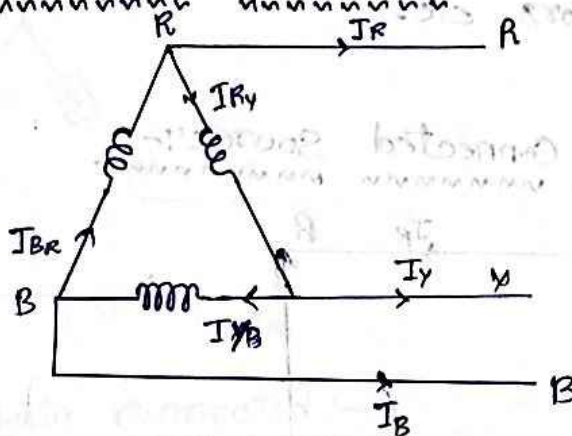
Three-phase three wire system!



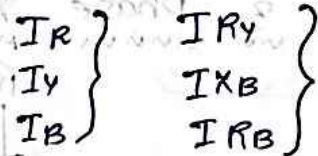
**3-φ four wire System:-**



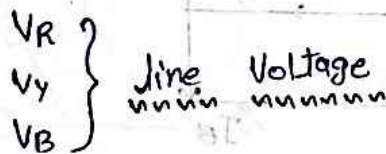
**Delta Connected System:-**



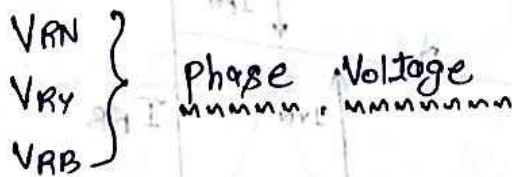
line current phase system:-



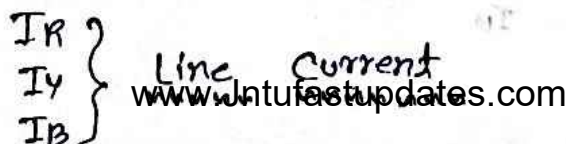
\* **Line Voltage:-** It is a voltage between two lines.



\* **Phase Voltage:-** The voltage across line and neutral is termed as phase voltage.



\* **Line current:-** The current following through each line is denoted as line currents.



phase current:- The current flowing through each phase is known as phase currents.

$I_{RY}$   
 $I_{YB}$   
 $I_{BR}$

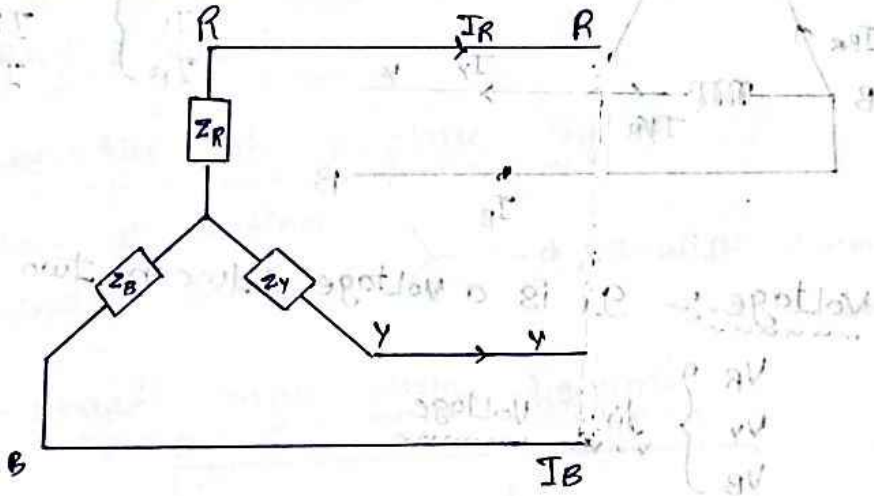
} phase currents

Source:- It is a source which generates power to the circuit.

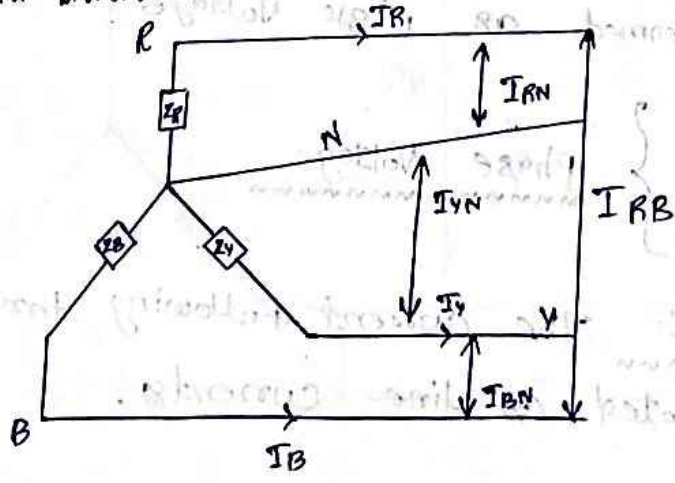
Ex:- Voltage Source  
 Current Source  
 Generators, etc.



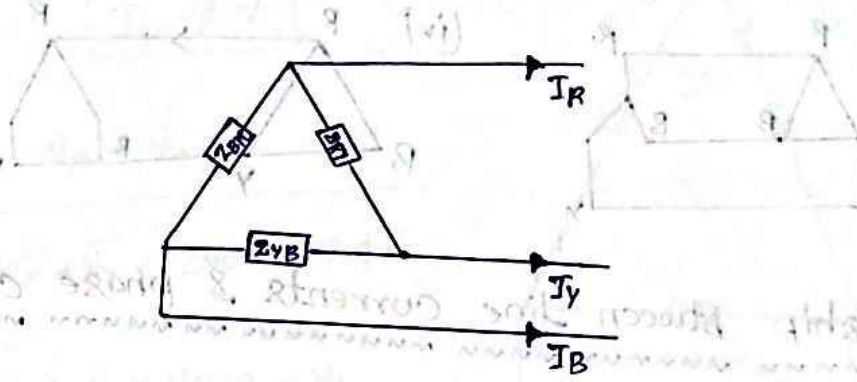
\* 3-phase star connected source:-



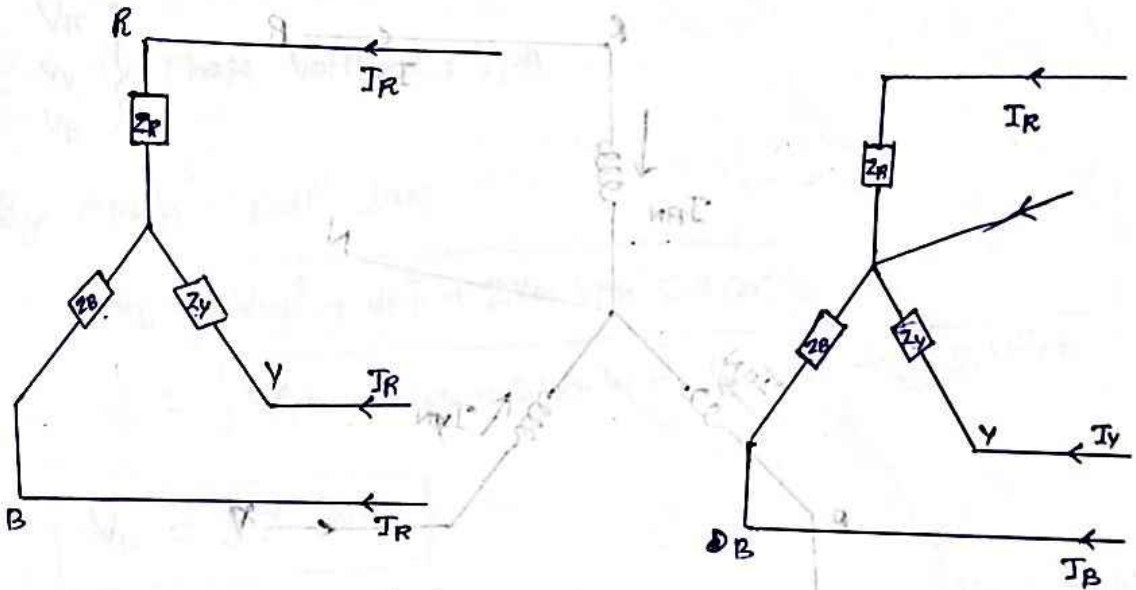
\* 3-φ 4 wire system:-



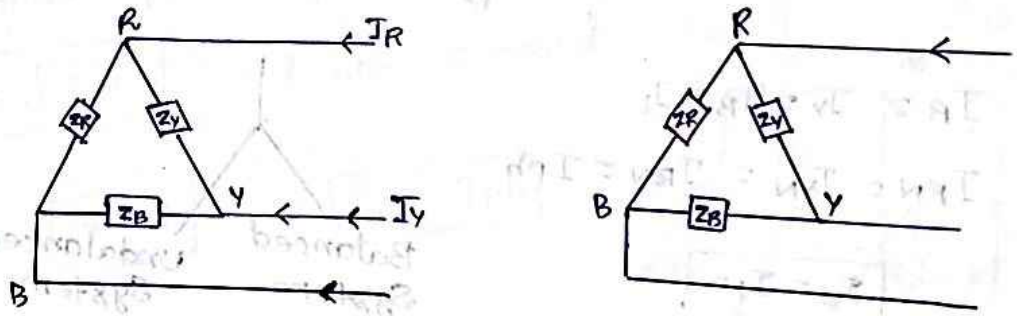
\* 3- $\phi$  delta Connected Source :-



\* Load :- It takes power from source, ex:- fans, lights etc.

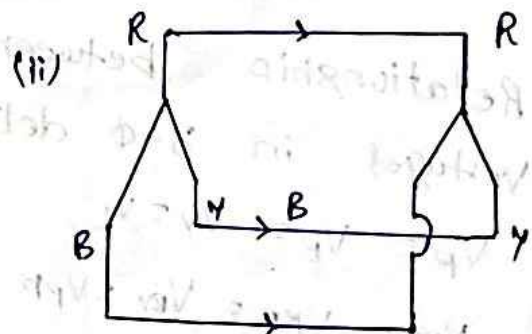
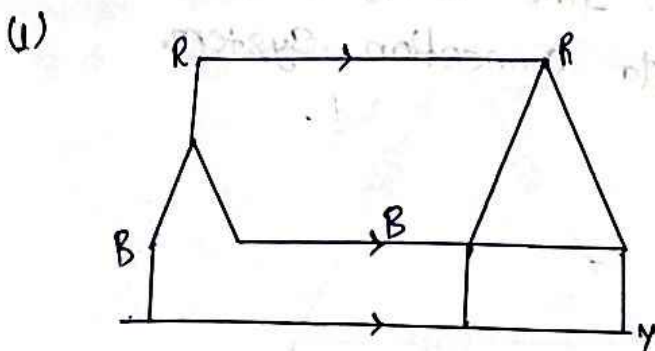


\* 3- $\phi$  delta connected :-

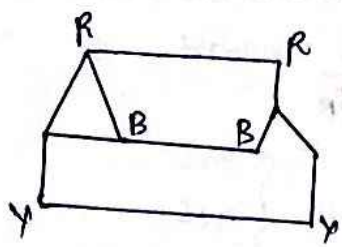


Examples for connection of

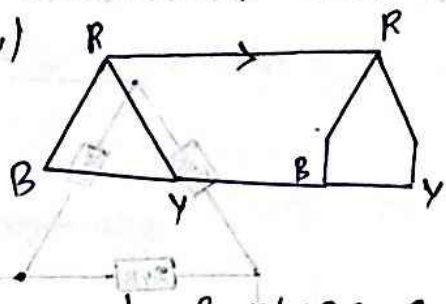
Source 3-load :-



(iii)

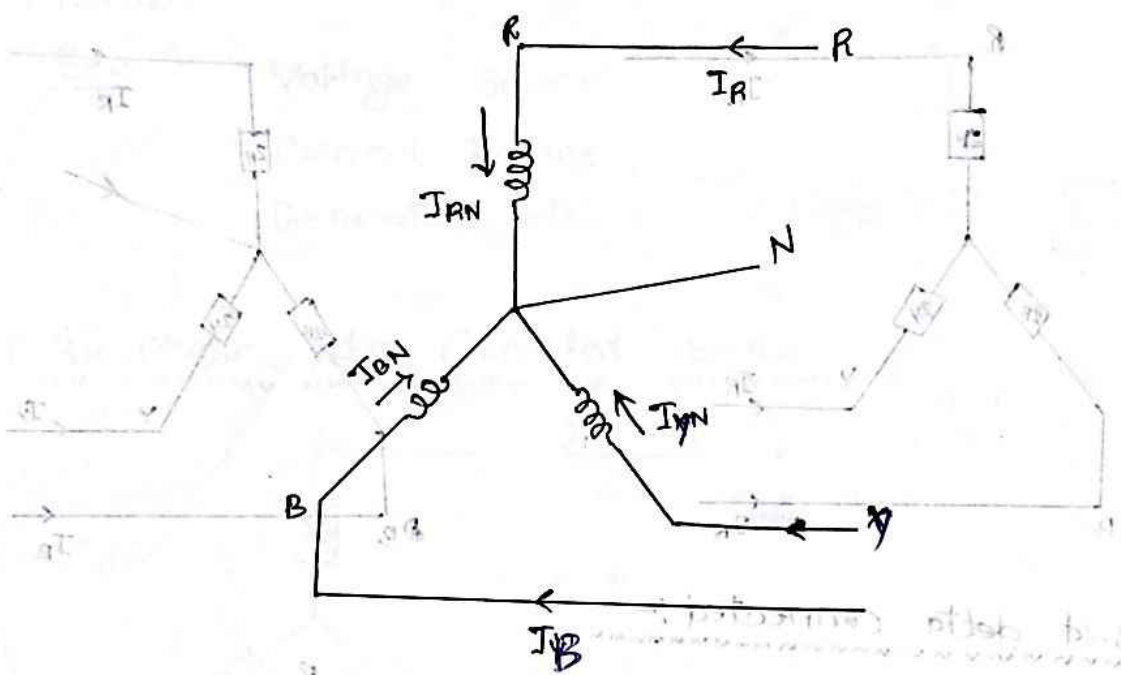


(iv)



\* Relationship between line currents & phase current :-

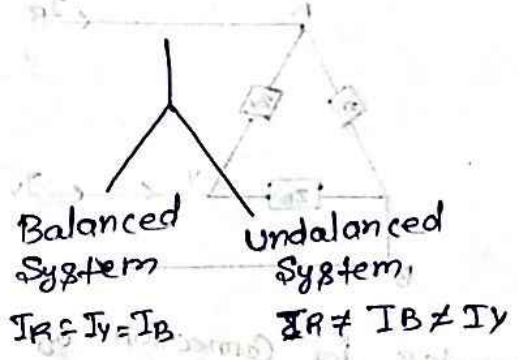
in 3- $\phi$ , 4-Wire System :-



$$I_R = I_Y = I_B = I_L$$

$$I_{RN} = I_{YN} = I_{BN} = I_{ph}$$

$$I_L = I_{ph}$$

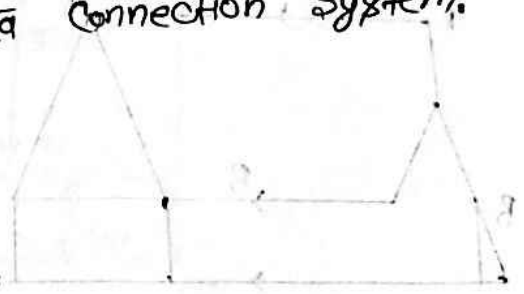


\* Relationship between line voltages & phase voltages in 3- $\phi$  delta connection system.

$$V_R = V_B = V_Y = V_L$$

$$V_{RY} = V_{RB} = V_{YB} = V_{ph}$$

$$V_L = V_{ph}$$



but

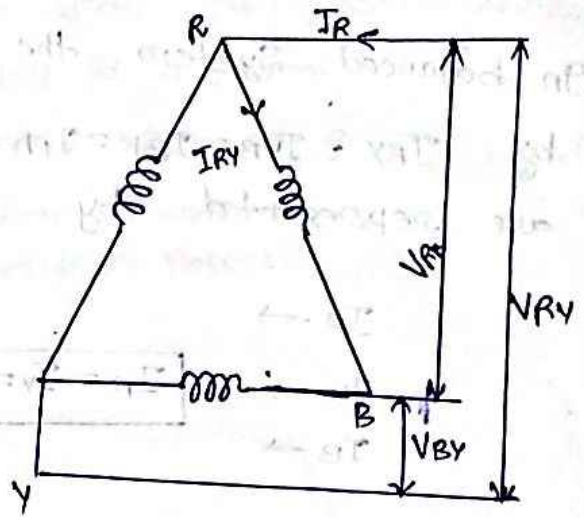
$$I_L \neq V_L$$

$$V_{RY} = \bar{V}_R - \bar{V}_Y$$

$$= \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos \phi}$$

$$V_{RY} = \text{line Voltage} = V_L$$

$$\left. \begin{matrix} V_R \\ V_Y \\ V_B \end{matrix} \right\} \text{Phase Voltage} = V_{ph}$$



By apply poll law

$$V_L = \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph} V_{ph} \cos 60^\circ}$$

$$= \sqrt{V_{ph}^2 + V_{ph}^2 + 2V_{ph} V_{ph} \cdot \frac{1}{2}} = \sqrt{3 V_{ph}^2}$$

$$V_L = \sqrt{3} V_{ph}$$

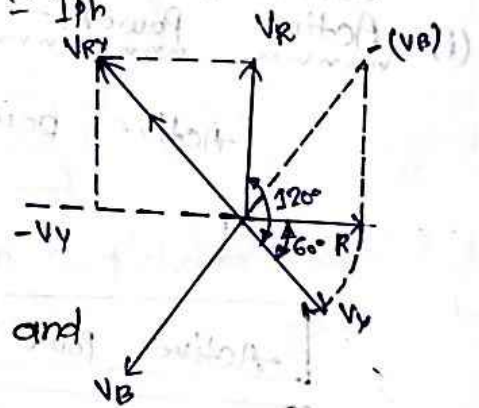
\* In three phase star connected system line Voltage

$$V_L = \sqrt{3} V_{ph}$$

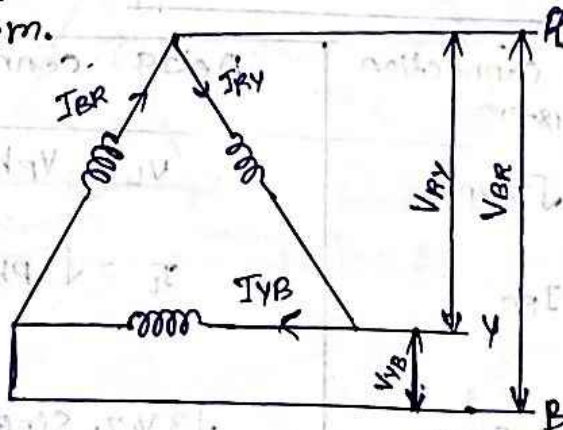
& line current =  $I_{ph}$

$$V_L = \sqrt{3} V_{ph}$$

$$I_L = I_{ph}$$



\* Relationship between line Voltages and phase Voltages in delta connected 3- $\phi$  System.



$$V_L = V_{ph}$$

$V_{RY}, V_{YB}, V_{BR} \rightarrow$  Line Voltage  
 $V_R, V_B, V_Y \rightarrow$  Phase Voltage  
 $I_{RY}, I_{YB}, I_{BR} \rightarrow$  Phase Currents

$$I_{RY} = I_{YB} = I_{BR} = I_{ph}$$

In balanced system the phase current is given by  $I_{RY} = I_{YB} = I_{BR} = I_{ph}$  and the line currents are represented by

$I_R \rightarrow$

$I_Y \rightarrow$

$I_B \rightarrow$

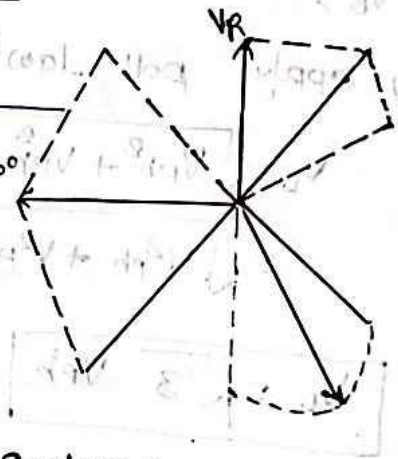
$$I_R = I_Y = I_B = I_L$$

\* Applying Parallelogram Law :-

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}I_{ph}\cos\theta}$$

$$I_L = \sqrt{I_{ph}^2 + I_{ph}^2 + 2I_{ph}^2\cos 60^\circ}$$

$$I_L = \sqrt{3} I_{ph}$$



\* Power in delta connected system :-

(i) Active Power :-

$$\text{Active power} = 3 \cdot V_{ph} \cos\phi$$

$$= 2 \times N_L \cdot \frac{I_L}{\sqrt{3}} \cos\phi$$

$$\text{Active Power} = \sqrt{3} V_L I_L \cos\phi$$

$$\text{Reactive Power} = \sqrt{3} V_L I_L \sin\phi$$

Relation between phase & Line	Star Connection System	Delta connected system
Voltage	$V_L = \sqrt{3} V_{ph}$	$V_L = V_{ph}$
Current	$I_L = I_{ph}$	$I_L = \sqrt{3} I_{ph}$
Active power		
Reactive power	$\sqrt{3} V_L I_L \cos\phi$	$\sqrt{3} V_L I_L \sin\phi$



\* A balanced 4-connected star  $2+j3$  supply is  
 hm per phase is connected to a balanced load.  
 3- $\phi$  440V supply the phase current is 10 AMP find (i) Total Active Power  
 (ii) Total Reactive Power

$\Rightarrow$  Active Power

$$\sqrt{3} V_L I_L \cos \phi$$

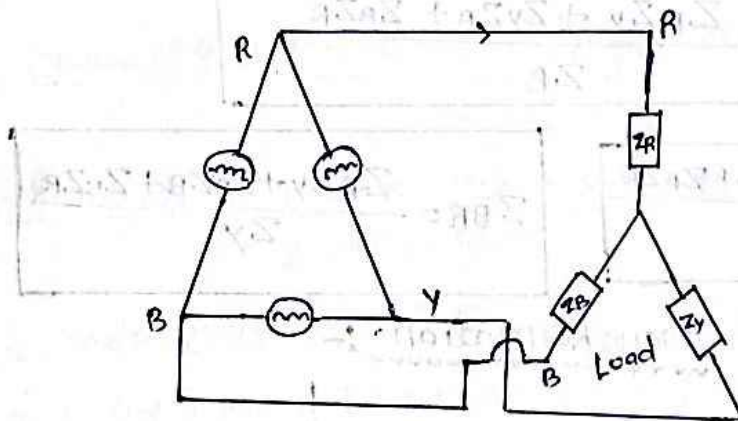
given

each phase  $Z_{ph} = 2+j3 = Y$  In  $\Delta I_L = \sqrt{3} I_{ph}$

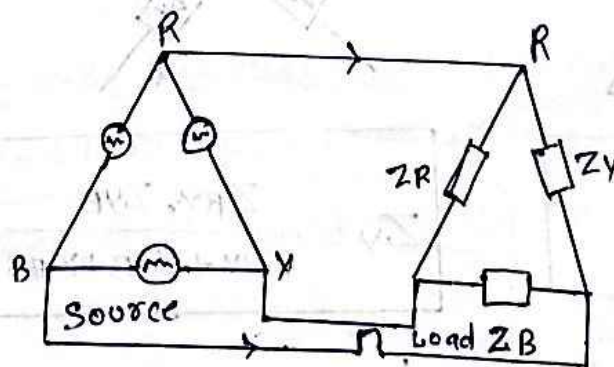
$$V_L = 440 \text{ V}$$

$$I_{ph} = 10$$

\* Draw the interconnection between a 3-phase  
 A-connected Source and a  $\lambda$ -connected Load.

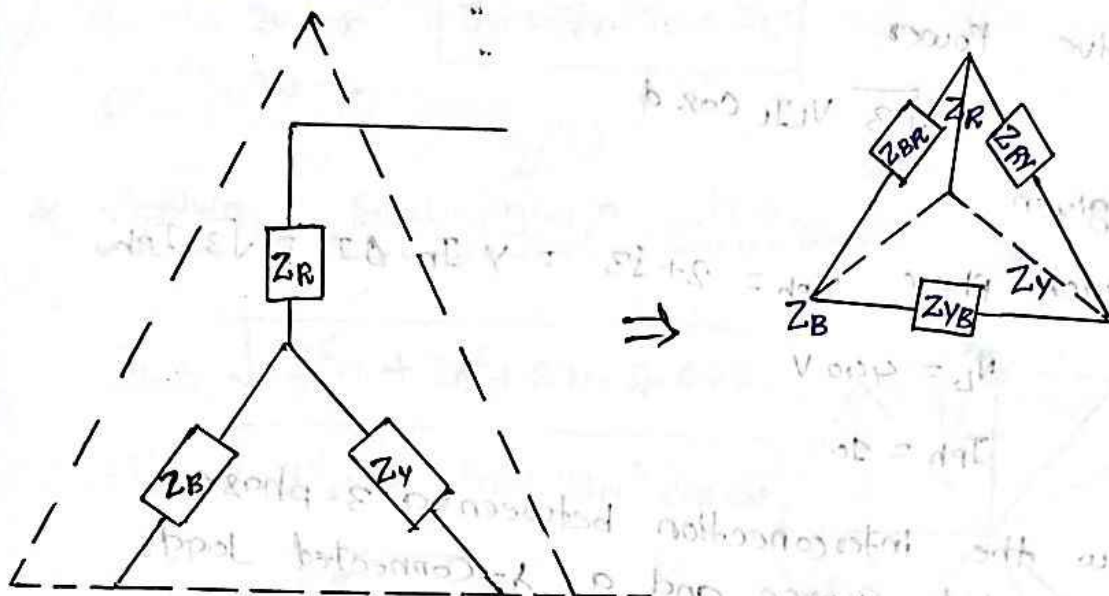


\* Draw the interconnection between 3- $\phi$  delta connected  
 Source and  $\Delta$ -connected load.



In balanced \* Star-delta 3-delta star transformation  
 by — => Let us consider a star connected system with  
 impedances of each phase is given by  $Z_R, Z_Y, Z_B$   
 and also consider  $\Delta$ -connected system with  
 impedance in each phase is given by

$Z_{RY}, Z_{YB}, Z_{BR}$



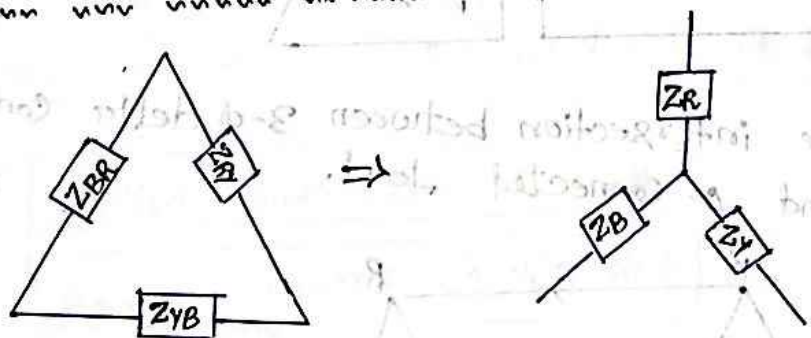
Star:-

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

\* Delta to star transformation :-



$$Z_R = \frac{Z_{RY} \cdot Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_Y = \frac{Z_{RY} \cdot Z_{YB}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$Z_B = \frac{Z_{YB} \cdot Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

\* A Symmetrical three phase 3-wire 440V supply is connected to star connected load the load impedance in each branch are,

Given data  $\Rightarrow$

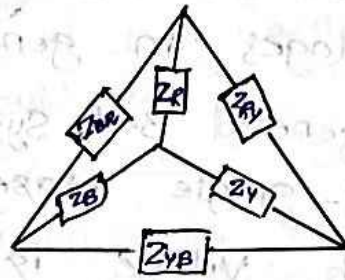
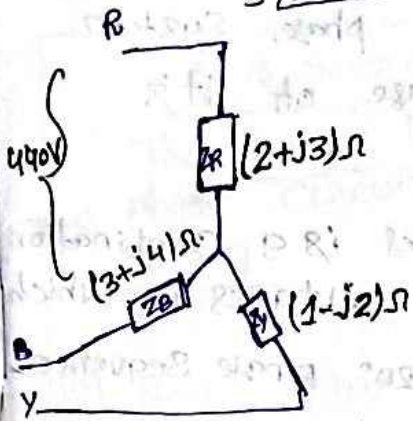
$$Z_R = \frac{2 + j3 \Omega}{3.606 \angle 56.30}$$

$$Z_Y = \frac{1 - j2 \Omega}{2.236 \angle -63.43}$$

$$Z_B = \frac{3 + j4 \Omega}{5 \angle 53.13}$$

Find its equal  $\Delta$ -connected load.

We have to convert, star connected load into  $\Delta$ -connected load.



for find  $Z_{BR}$ ,  $Z_{RY}$ ,  $Z_{YB}$ ,

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y} = \frac{(2 + j3)(1 - j2) + (1 - j2)(3 + j4) + (3 + j4)(2 + j3)}{1 - j2}$$

$$Z_{BR} = \frac{(3.606 \angle 56.30)(2.236 \angle -63.43) + (2.236 \angle -63.43)(5 \angle 53.13) + (5 \angle 53.13)(3.606 \angle 56.30)}{2.236 \angle -63.43}$$

$$Z_{BR} = \frac{8.063 \angle -7.13 + 11.18 \angle -10.3 + 18.03 \angle 109.43}{2.236 \angle -63.43}$$

$$Z_{BR} = 3.606 \angle 56.3 + 5 \angle 53.13 + 8.064 \angle 172.86$$

$$Z_{BR} = 2.0 + j3.0 + 3.0 + j3.9 - 8.0 + j1.0$$

$$Z_{BR} = -3 + j7.9$$

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} = \frac{8.063 \angle -7.13 + 11.18 \angle -10.3 + 18.03 \angle 109.43}{5 \angle 53.13}$$

$$Z_{RY} = 1.6126 \angle -60.26 + 2.236 \angle -63.43 + 3.606 \angle 56.3$$

$$0.799 - j1.40 + 1.0 - j1.99 + 2.00 + j3.00$$

$$Z_{RY} = 3.799 - j0.39 \text{ A}$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R} = \frac{8.063 \angle -7.13 + 11.18 \angle -10.3 + 18.03 \angle 109.43}{3.606 \angle 56.30}$$

$$Z_{YB} = 2.236 \angle -63.43 + 3.10 \angle -66.8 + 5 \angle 53.13$$

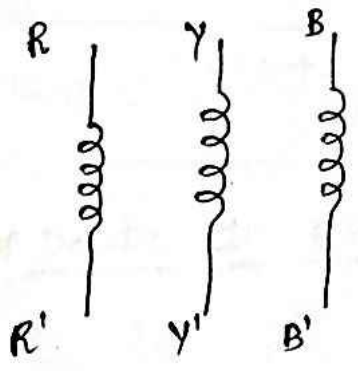
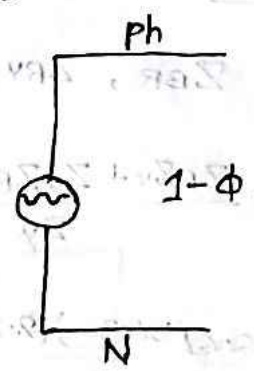
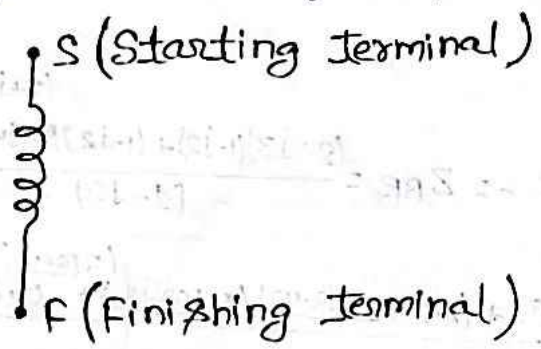
$$1.0 - j1.99 + 1.0 - j1.99 + 1.0 - j1.99$$

$$Z_{YB} = 5.23 - j0.84 \quad \text{Ans}$$

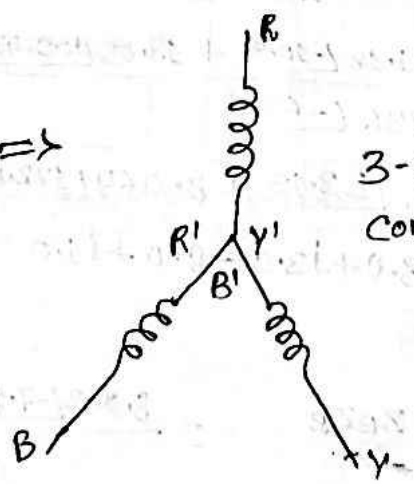
\* The Value of current Balanced 3- $\phi$  System:-

$\Rightarrow$  In one A.C System we connect two or more number of individual circuits two a common poly phase source through it is possible to have any number of sources in poly phase system but we use 3- $\phi$  system because of its advantages in general.

$\Rightarrow$  In general 3- $\phi$  system of voltages is a combination of 3- single phase system of voltages in which each voltage is differ by  $120^\circ$  phase sequences

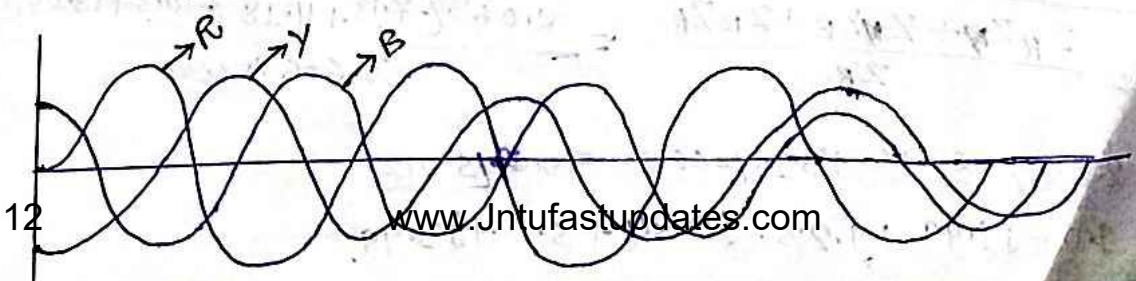
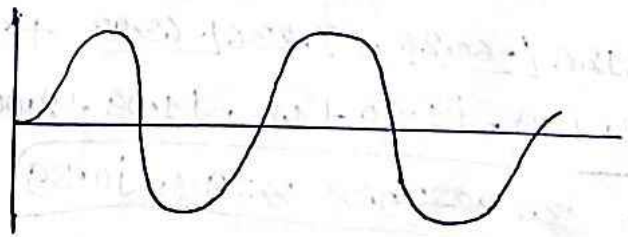


$\Rightarrow$



3- single phase or connected in one = 3- $\phi$  By.

Single Phase:-



\* Advantages of 3- $\phi$  Single when Compare to 1- $\phi$  System.

=> Based on usage point of view and Consumer point of view 3- $\phi$  system has been many advantages when Compare to single-phase System.

1- $\phi$ System	3- $\phi$ System
[1] The power in single phase circuit is <u>pulse rating</u> .	[1] The total 3- $\phi$ power supplied to a balanced 3- $\phi$ circuit is <u>constant at every instant of time</u> .
[2] Single phase torque produced in single phase motor pulse rating term.	[2] Because of the constant power 3- $\phi$ motors <u>have uniform torque</u> .
[3] The output of single phase motor is <u>less</u> .	[3] 3- $\phi$ phase motor or 3- $\phi$ generator produces <u>more output</u> when Compare to 1- $\phi$ System.
[4] The transmit power in single phase system we require <u>large amount of conductor material</u> .	[4] 3- $\phi$ transmissions are give <u>require less conductor material</u> than single phase circuit.
[5] Single phase motors are <u>not self starting</u> where as	[5] 3- $\phi$ motors are <u>easily started</u> .

The sequence of voltage in 3- $\phi$ 's are in the order. The sequence of attaining maximum values by three phase voltage (or) current is known as phase sequence.

$$V_{RR'} - V_{YY'} - V_{BB'}$$

-> These order of the system is known as phase sequence.

$$V_R = V_m \sin \omega t$$

$$V_Y = V_m \sin (\omega t - 120^\circ)$$

$$V_B = V_m \sin (\omega t - 240^\circ)$$

\* There are two types of phase Sequences

[i] Positive phase Sequence

[ii] Negative phase Sequence.

→ In positive phase Sequence the field is rotated in clock wise direction.



→ RYB phase Sequence

Clock Wise

The Voltage in each phase is given by  $V_R = V_m \sin \omega t$

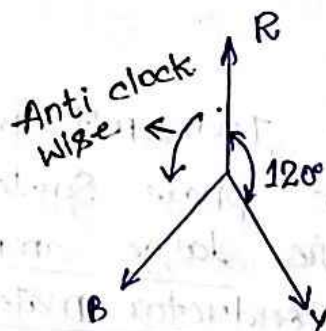
$$V_Y = V_m \sin(\omega t - 120^\circ)$$

$$V_B = V_m \sin(\omega t - 240^\circ)$$

Negative Phase Sequence:-

\* Here the field is Voltage in Anti clock wise direction and

\* The phase Sequence is RYB Sequence



RYB

\* Each phase is given by,

$$V_R = V_m \sin \omega t$$

$$V_B = V_m \sin(\omega t - 120^\circ)$$

$$V_Y = V_m \sin(\omega t - 240^\circ)$$

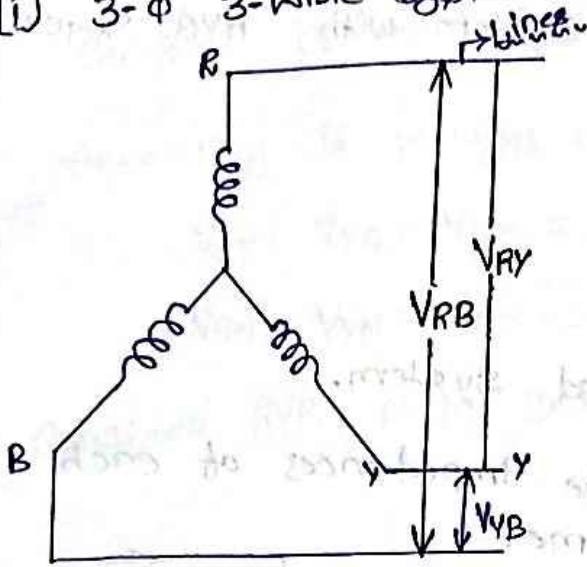
\* Types of Connection (or) Interconnection of 3- $\phi$  System:-

[i] Star Connected System,

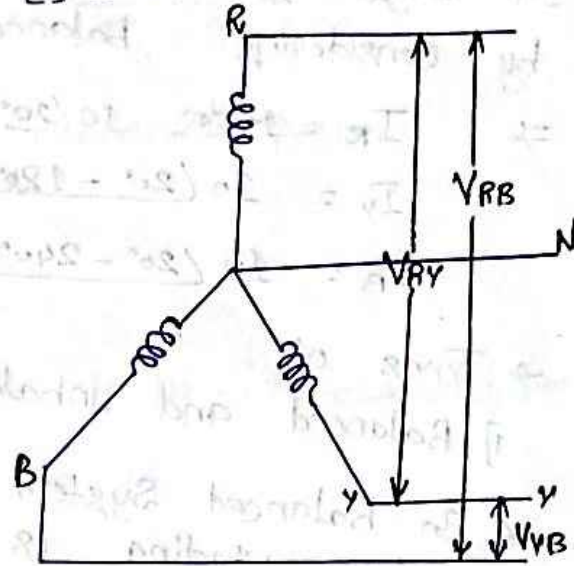
[ii] Delta Connected System.

\* Star Connected System:-

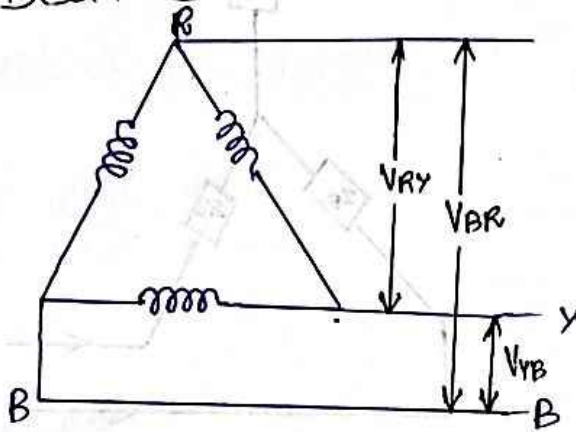
[i] 3- $\phi$  3-wire System



[ii] 3- $\phi$ , 4-wire System,

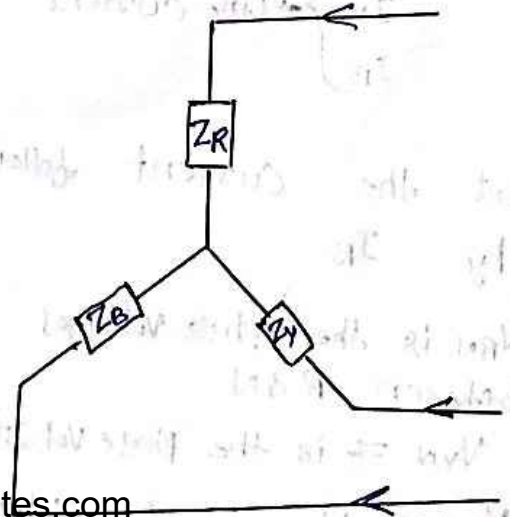
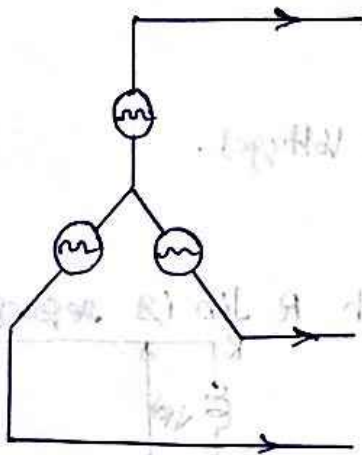


\* Delta Connected System:-



\* Source:- It delivers power to the circuit that means give energy to the circuit. Ex:- Voltage Source, current Source, Generator Source.

\* Load:- It takes power from sources. Ex:- fans, light, Coolers etc.



Problem:- The value of the current in phase or is at an angle  $20^\circ$ . Calculate the values of 3-line currents by considering balanced system with RYB sequence.

$$\Rightarrow I_R = 10 \angle 20^\circ$$

$$I_Y = 10 \angle 20^\circ - 120^\circ$$

$$I_B = 10 \angle 20^\circ - 240^\circ$$

→ Types of  
1) Balanced and unbalanced system.

\* In balanced system the impedances of each phase winding is same.

$$Z_R = Z_Y = Z_B$$

$$|Z_R| < \phi$$

$$|Z_R| = |Z_Y| = |Z_B|$$

\* Unbalanced system:-  
The phase winding for each phase different is

$$Z_R \neq Z_Y \neq Z_B$$

\* Relation between line voltages and phase voltage in star-connected system.

⇒ Let us consider a star connected system (balanced)

Let  $I_R$  }  
 $I_Y$  } → Line current  
 $I_B$  }

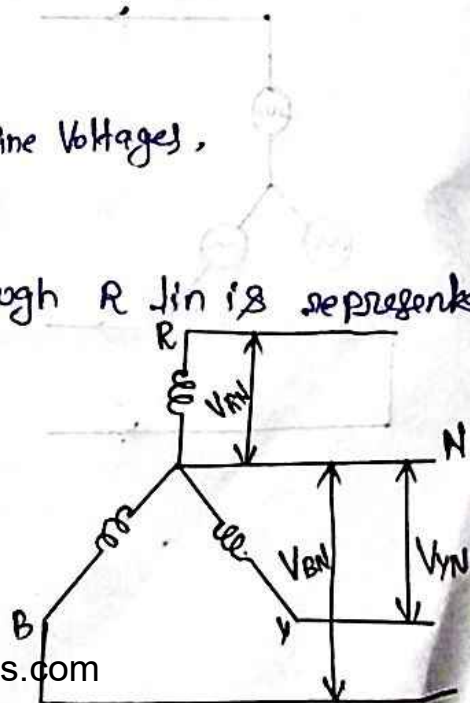
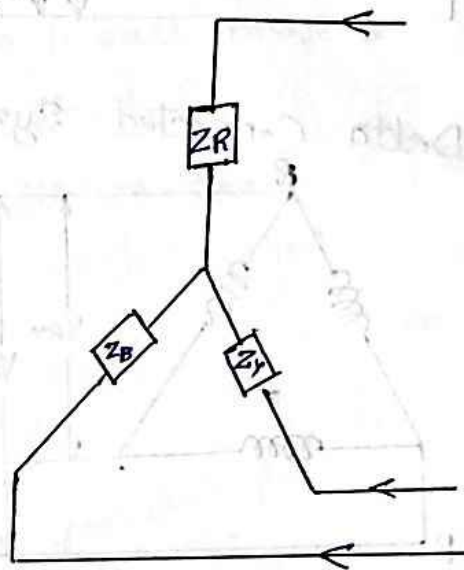
$V_R$  }  
 $V_Y$  } → Line Voltages,  
 $V_B$  }

\* Let the current flowing through R in is represented by  $I_R$

$V_{RN}$  is the phase voltages between R & N

$V_{YN}$  is the phase vol btw Y & N

$V_{BN}$  is the phase vol btw B & N





Here Considering balanced types of system.

$$I_R = I_Y = I_B = I_L$$

In star connected system,

$$I_L = I_{ph}$$

\* According to voltages in  $\gamma$ -connected system.

$$V_{RY} = V_{YB} = V_{BR} = V_L$$

$$V_{RN} = V_{YN} = V_{BN} = V_{ph}$$

\* Consider RYB phase sequence in phasor representation

\* A 3-phase 3 wire 220 V 440 Volts supply is applied to balanced with 3-phase star connected load  $20 \angle 30^\circ$  at an angle  $30^\circ$  ( $20 \angle 30^\circ$ ) in each phase find  $R + jX_L$

- (i) line voltages
- (ii) Phase voltages
- (iii) phase currents
- (iv) Line currents

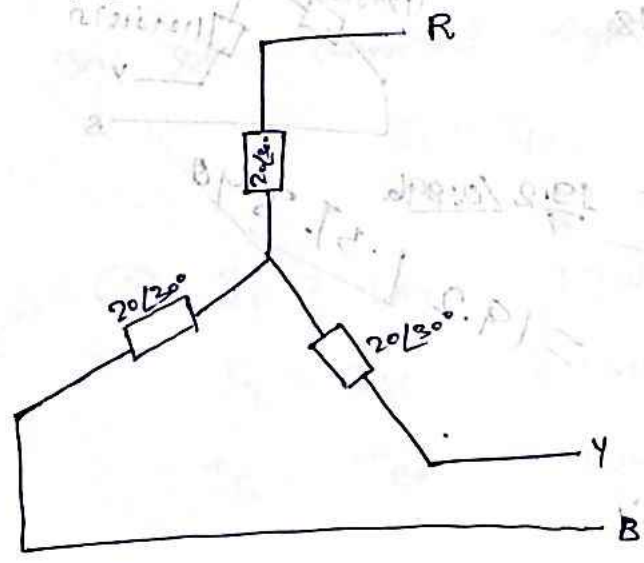
By assuming ~~are~~ RYB phase sequence.  $R \rightarrow j \rightarrow c$

= Given data :

(V<sub>L</sub>) Line Voltages = 440 V

3- $\phi$  Star Connection

$Z_R$   
 $Z_Y$  }  $20 \angle 30^\circ$   
 $Z_B$



(i) line voltages

$$V_{RY} = 440 \angle 0^\circ$$

$$V_{YB} = 440 \angle -120^\circ$$

$$V_{BR} = 440 \angle -240^\circ$$

(ii) phase voltages

$$V_L = \sqrt{3} V_{ph} = V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$V_{RN} = \frac{440 \angle 0^\circ}{\sqrt{3}}$$

$$V_{YN} = \frac{440 \angle -120^\circ}{\sqrt{3}}$$

$$V_{BN} = \frac{440 \angle -240^\circ}{\sqrt{3}}$$

(iii) ~~line~~ current =

(iii) Phase currents

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{254.09 \angle -240^\circ}{20 \angle 30^\circ} = 12.70 \angle -270^\circ$$

$$\frac{254.09 \angle -240^\circ}{20 \angle 30^\circ} = 12.70 \angle -270^\circ$$

$$I_R = \frac{V_{RN}}{Z_R} = \frac{254.09 \angle 0^\circ}{20 \angle 30^\circ} = 12.70 \angle -270^\circ$$

$$I_y = \frac{V_{YN}}{Z_y} = \frac{254.09 \angle -120^\circ}{20 \angle 30^\circ} = 12.70 \angle -150^\circ$$

line Current

$$I_L = I_{ph}$$



A 3-phase 3-wire 415 Volt supply is applied to a balanced 3-phase star connected load  $12 + j15 \Omega$  in each phase find (i) line Voltages

(ii) Phase Voltages (iii) Phase Current (iv) Phase Voltages

By assuming RYB phase Sequence

⇒ Given data :-

Line Voltage = 415 Volt

$$\left. \begin{matrix} Z_R \\ Z_Y \\ Z_B \end{matrix} \right\} = 12 + j15 = 19.2 \angle 0.996$$

line Phase Voltages =

$$V_{RY} = 415 \angle 0^\circ$$

$$V_{YB} = 415 \angle -120^\circ$$

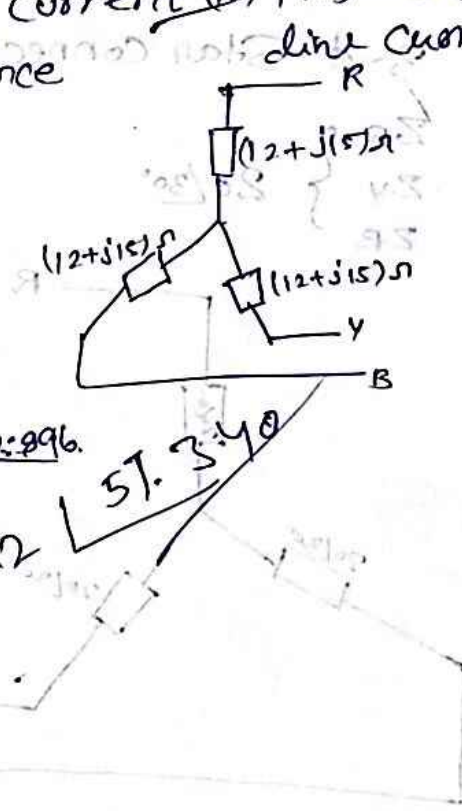
$$V_{BR} = 415 \angle -240^\circ$$

$$\text{phase Voltage} = V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{415 \angle 0^\circ}{\sqrt{3}} = 239.6 \angle 0^\circ$$

$$V_{RY} = \frac{415 \angle 0^\circ}{\sqrt{3}} = 239.6 \angle 0^\circ$$

$$V_{YN} = \frac{415 \angle -120^\circ}{\sqrt{3}} = 239.6 \angle -120^\circ$$

$$V_{BN} = \frac{415 \angle -240^\circ}{\sqrt{3}} = 239.6 \angle -240^\circ$$



Phase Current

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

$$\frac{239.6 \angle 0^\circ}{19.2 \angle -51.34^\circ}$$

$$I_{RY} = \frac{V_{RN}}{Z_R} = \frac{239.6 \angle 0^\circ}{12 + j15}$$

$$\frac{239.6 \angle 0^\circ}{19.2 \angle -51.34^\circ}$$

$$12.47 \angle -51.34^\circ$$

$$I_{RY} = 12.47 \angle -51.34^\circ$$

$$I_{YB} = \frac{V_{YN}}{Z_Y} = \frac{239.6 \angle -120^\circ}{19.2 \angle -51.34^\circ} = 12.47 \angle -68.66^\circ$$

$$I_{BR} = \frac{V_{BN}}{Z_B} = \frac{239.6 \angle -240^\circ}{19.2 \angle -51.34^\circ} = 12.47 \angle -188.66^\circ$$

Line Current:

In star connected system

$$I_L = I_{ph}$$

$$I_R = I_{RY} = 12.47 \angle -51.34^\circ$$

$$I_Y = I_{YB} = 12.47 \angle -68.66^\circ$$

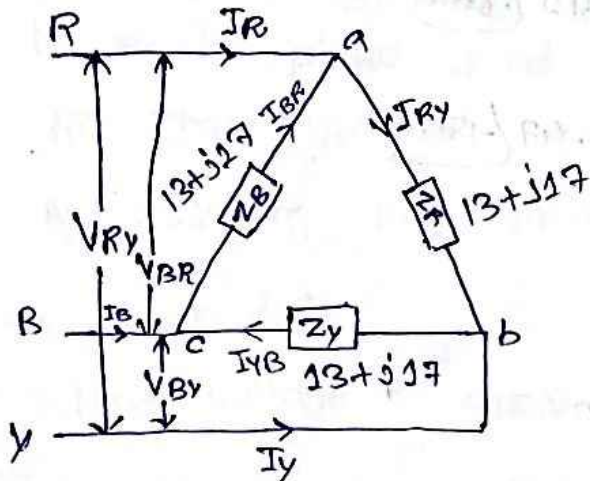
$$I_B = I_{BR} = 12.47 \angle -188.66^\circ$$



\* A balanced  $\Delta$ -Delta Connected system has a load of  $13 + j17 \Omega$  in each phase is connected to 3-phase 400 Volt  $\Delta$ -Delta Connected source. Calculate

- (i) Line Voltages      (iii) Phase Current  
 (ii) Phase Voltages    (iv) Line Current

Assume RYB phase sequence.



[i] Line Voltages

$$V_{RY} = 400 \angle 0^\circ$$

$$V_{YB} = 400 \angle -120^\circ$$

$$V_{BR} = 400 \angle -240^\circ$$

[ii] Phase Voltages

In delta-connected system

$$V_L = V_{ph}$$

Phase - Voltages

$$\begin{cases} V_R = 400 \angle 0^\circ \\ V_Y = 400 \angle -120^\circ \\ V_B = 400 \angle -240^\circ \end{cases}$$

$$11.43 - j14.96$$

$$7.17 + j17.25$$

$$4.26 + j2.49$$

(iii)  $\left. \begin{matrix} I_{RY} \\ I_{YB} \\ I_{BR} \end{matrix} \right\}$  phase current

$$I_{ph} = \frac{V_{ph}}{Z_{ph}}$$

$$I_{RY} = \frac{V_{RY}}{Z_R} = \frac{400 \angle 0^\circ}{13 + j17} = \frac{400 \angle 0^\circ}{21.4 \angle 52.59^\circ} =$$

$$I_{YB} = \frac{V_{YB}}{Z_Y} = \frac{400 \angle -120^\circ}{13 + j17} = \frac{400 \angle 120^\circ}{21.4 \angle 52.59^\circ}$$

$$= 18.69 \angle -172.59^\circ$$

$$I_{BR} = \frac{V_{BR}}{Z_B} = \frac{400 \angle -240^\circ}{21.4 \angle 52.59^\circ} = 18.69 \angle -292.59^\circ$$

$$= 18.69 \angle -72.59^\circ$$

(iv) Line Current:-

$I_R$   
 $I_Y$   
 $I_B$

} Line Currents

At node a, by applying KCL

$$I_R + I_{BR} = I_{RY}$$

$$I_R = I_{RY} - I_{BR} = I_R = 18.69 \angle -52.59^\circ - 18.69 \angle -292.59^\circ$$

$$I_R = 32.36 \angle -82.59^\circ \text{ A}$$

At node b applying KCL

$$I_{RY} + I_Y = I_{YB}$$

$$18.69 \angle -52.59^\circ + I_Y = 18.69 \angle -172.59^\circ = I_Y$$

$$12.667 \angle -11.16^\circ - j13.742 + 16.984 \angle -172.59^\circ = I_Y$$

$$29.651 - j21.542$$

$$= 36.650 \angle -39.398^\circ$$

At node c applying KCL

$$I_B + I_{YB} = I_{BR} = I_B = I_{BR} - I_{YB}$$

$$I_B = 18.69 \angle -292.59^\circ - 18.69 \angle -172.59^\circ =$$

$$7.179 + j17.256 + 18.534 \angle -172.59^\circ - j2.410$$

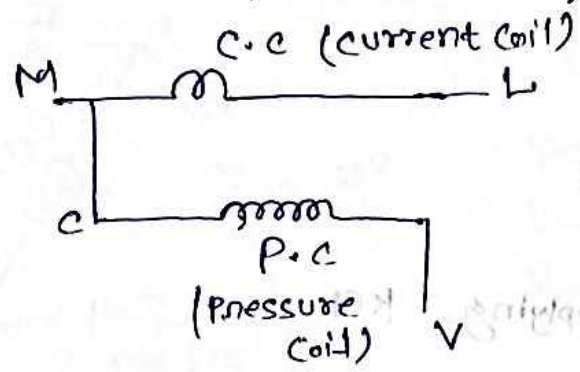
$$25.713 + j14.846 = 29.691 \angle 30.00^\circ$$

\* Measurement of power in 3- $\phi$  circuit:-

- Measurement of power in any circuit is wattmeter
- Measurement of current use ammeter
- Measurement of voltage use voltmeter

Wattmeter

- (i) Current coil
- (ii) Pressure coil / Voltage coil / Potential coil.

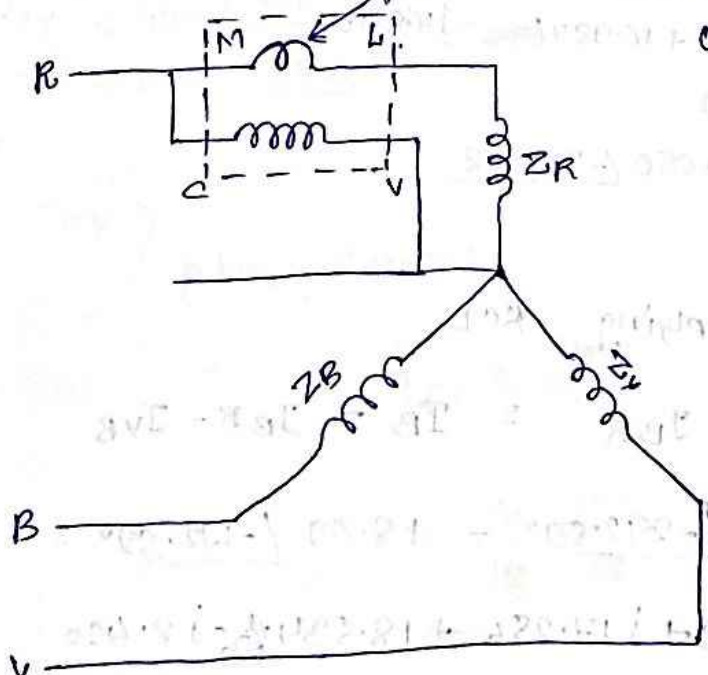


Where  
 M = Main Side  
 L = Load Side  
 C = Common Side  
 V = Voltage Side

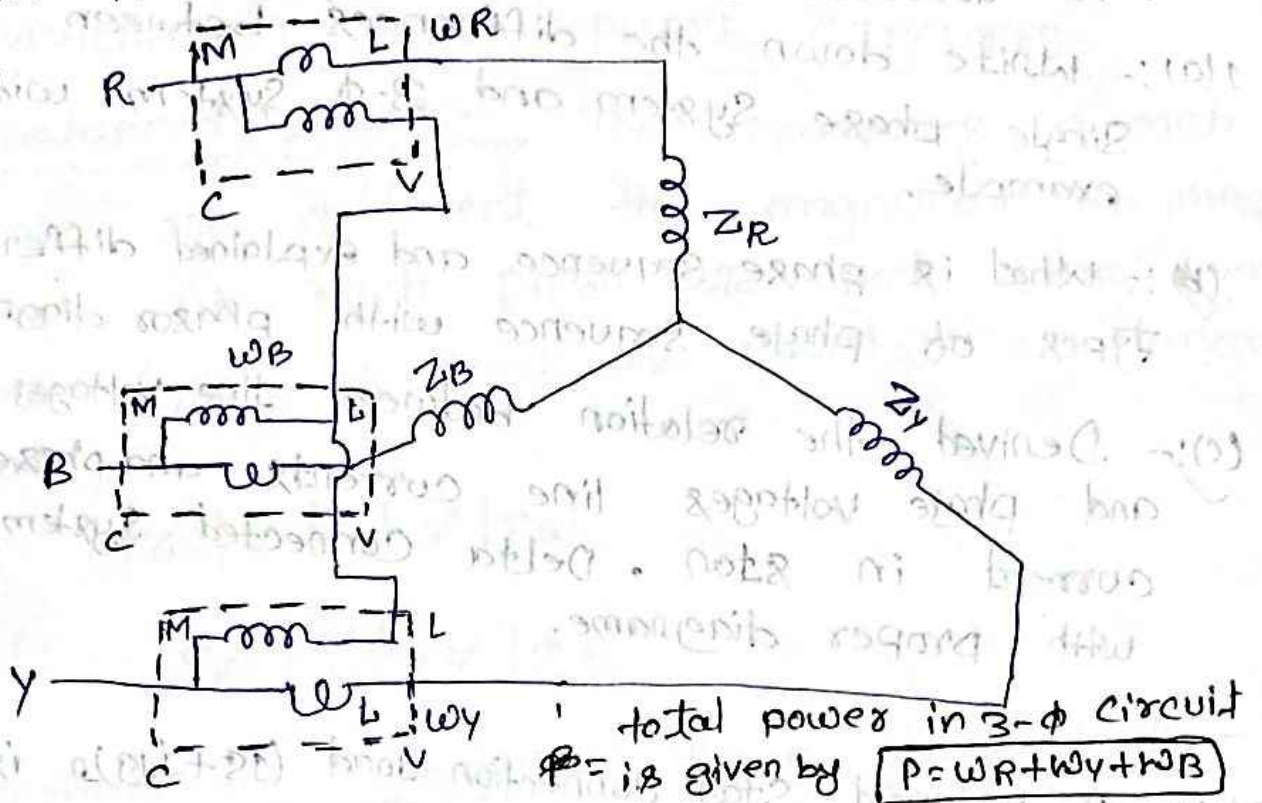
To measure power in 3- $\phi$  circuit is

- (i) Single wattmeter Method
- (ii) Two wattmeter method
- (iii) Three wattmeter method.

\* Single wattmeter method:- Method is used for balanced types of loads only.



\* ~~Two~~ Three Wattmeter method :-



Where  $W_R$ ,  $W_Y$ ,  $W_B$  is the watt meter reading which is connected in R line ( $W_R$ )  
 $W_Y$  is the watt meter <sup>reading</sup> which is connected in Y line.  
 $W_B$  is the watt meter reading which is connected in B line.

Calculate (i) the real power and reactive power in the circuit  
 (ii) the power factor  
 (iii) the complex power  
 (iv) the average power  
 (v) the complex power  
 (vi) the real power  
 (vii) the reactive power  
 (viii) the complex power  
 (ix) the real power  
 (x) the reactive power  
 (xi) the complex power  
 (xii) the real power  
 (xiii) the reactive power  
 (xiv) the complex power  
 (xv) the real power  
 (xvi) the reactive power  
 (xvii) the complex power  
 (xviii) the real power  
 (xix) the reactive power  
 (xx) the complex power



## VVI Question from unit - 1

1(a) :- Write down the differences between Single phase System and 3- $\phi$  System with example.

(b) :- What is phase sequence and explained different types of phase sequence with phasor diagram.

(c) :- Derived the relation between line voltages and phase voltages line currents and phase current in star, Delta connected system with proper diagram.

2(a) :- A balanced star connection load  $(15 + j19)\Omega$  is connected in each phase to a 400V supply. Calculate (i) line voltages, (ii) phase voltages, (iii) phase current, (iv) line current by assuming RYB phase sequence.

(b) :- The power consumed in a 3- $\phi$  balanced  $\Delta$ -delta connected load is ~~2~~ 5 kW at a power factor 0.8 lagging. The supply voltage is 450V 50Hz. Calculate (i) The resistance and reactance of each phase, (ii) Lagging currents and phase current.

(c) :- How can we measure the power in 3- $\phi$  circuit? What are the different methods to measure power.

## UNBALANCED THREE PHASE CIRCUITS

\* Unbalanced loads! - The impedances in each phase are different the magnitude and phase angle of each phase are not equal then this type of load are called as unbalanced load.

$$|Z_R| \neq |Z_Y| \neq |Z_B|$$

$$\angle \phi_R \neq \angle \phi_Y \neq \angle \phi_B$$

\* Analysis of unbalanced three phase loads! -

(i) Analysis of unbalanced 3- $\phi$   $\Delta$ -Delta Connected Load.

(ii) Analysis of unbalanced 3- $\phi$  star Connected Load.

\* Analysis of unbalanced 3- $\phi$  Delta Connected Load.

$\Rightarrow$  Consider An unbalanced  $\Delta$ -Delta Connected Load which is connected to a balanced 3- $\phi$  source. The impedances of unbalanced load are  $Z_{RY} \angle \phi_{RY}$ ,  $Z_{YB} \angle \phi_{YB}$ ,  $Z_{BR} \angle \phi_{BR}$

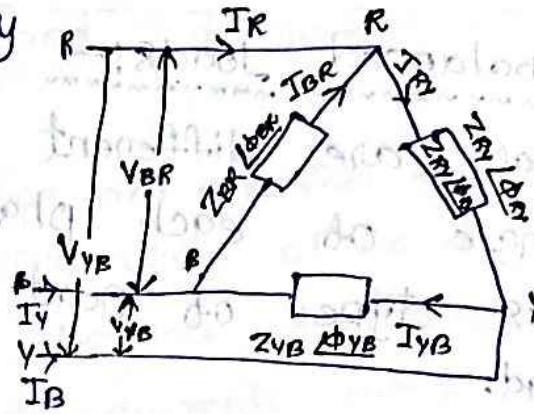
The unbalanced 3- $\phi$  Delta Connected Load is connected to balanced type of source hence the voltages between source and load are same

\* Line Voltages:- The line voltages in unbalanced system is given by

$$V_{RY} = V_L \angle 0^\circ$$

$$V_{YB} = V_L \angle -120^\circ$$

$$V_{BR} = V_L \angle -240^\circ$$



In unbalanced type of load the line currents and phase currents are different.

\* Phase Voltages:- The phase voltages in unbalanced system is given by.

In delta connected,

line voltages = phase voltage

$$V_{RY} = \text{line voltage}$$

$$V_{YB} = \text{"}$$

$$V_{BR} = \text{"}$$

\* phase currents:-

$$I_{RY}, I_{YB}, I_{BR}$$

$$I_{RY} = \frac{\text{Phase Voltage}}{\text{Phase impedance}} = \frac{V_{RY}}{Z_{RY}}$$

$$I_{YB} = \frac{\text{Phase Voltage}}{\text{Phase impedance}} = \frac{V_{YB}}{Z_{YB}}$$

$$27 \quad I_{BR} = \frac{V_{BR}}{Z_{BR}} \quad \text{www.Jntufastupdates.com}$$

Line Current:- The line currents are given by applying KCL at R

$$I_R + I_{BR} = I_{RY}$$

$$I_R = I_{RY} - I_{BR}$$

$$I_Y = I_{YB} - I_{RY}$$

$$I_B = I_{BR} - I_{YB}$$

\* In unbalanced 3- $\phi$   $\Delta$ -connected load has impedances  $10 \angle 20^\circ$ ,  $30 \angle 40^\circ$ ,  $10 \angle -90^\circ$  are connected to 400V 3- $\phi$  source then calculate

- (i) Line Voltages (ii) Phase Voltages (iii) phase current (iv) Line Currents.

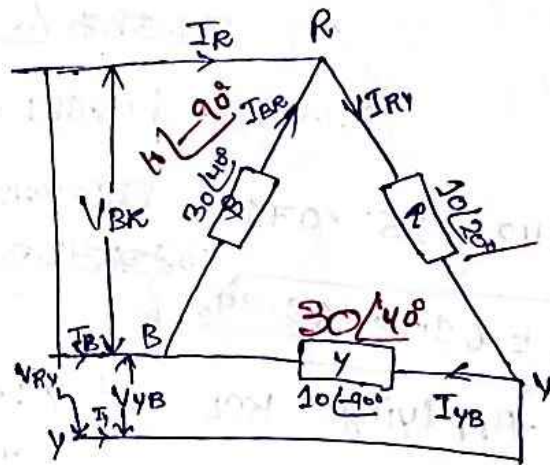
$\Rightarrow$  Given data

$$Z_{RY} = 10 \angle 20^\circ \quad V_L = 400V$$

$$Z_{BR} = 30 \angle 40^\circ$$

$$Z_{YB} = 10 \angle -90^\circ$$

(i) Line Voltages:-



$$* V_{RY} = V_L \angle 0^\circ = 400 \angle 0^\circ = 400V$$

$$V_{YB} = V_L \angle -120^\circ = 400 \angle -120^\circ$$

$$V_{BR} = V_L \angle -240^\circ = 400 \angle -240^\circ$$

(ii) Phase Voltages: In  $\Delta$ -Connection System Line voltages is equal to phase voltage.

$$\text{So, } V_{RY} = V_R = 400 \angle 0^\circ$$

$$V_{YB} = V_Y = 400 \angle -120^\circ$$

$$V_{BR} = V_B = 400 \angle -240^\circ$$

\* Phase Currents:  $I_{RY}, I_{YB}, I_{BR}$

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{400 \angle 0^\circ}{10 \angle 20^\circ} = 40 \angle -20^\circ$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{400 \angle -120^\circ}{10 \angle -90^\circ} = 40 \angle -30^\circ$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{400 \angle -240^\circ}{10 \angle -90^\circ} = 40 \angle -280^\circ$$

\* Line Current:-

Applying KCL at R,  $I_A = I_{RY} - I_{BR}$

$$I_R = 40 \angle -20^\circ - 40 \angle -280^\circ$$

$$37.587 - j13.681 - (6.945 + j39.39)$$

$$30.642 - j53.071$$

$$37.587 - j13.681 - (6.945 + j39.39) = 30.642 - j53.071$$

$$I_R = 61.28 \angle 59.998^\circ$$

Applying KCL at Y:-

$$I_Y = I_{YB} - I_{RY}$$

$$I_Y = 13.33 \angle -160^\circ - 40 \angle -20^\circ$$

$$-12.53 - j4.56 - 37.588 + j13.681$$

$$-50.118 + j9.121 = 50.94 \angle 169.68^\circ$$

$$34.844 - j26 - 37.587 + j13.680$$

$$-2.946 - j12.32 = 13.33 \angle -280^\circ$$

Applying KCL at B:-

$$I_B = I_{BR} - I_{YB}$$

$$I_B = 13.33 \angle -280^\circ - 40 \angle -30^\circ$$

~~$2.3782 \angle 134.27^\circ - 34.691 + j20$~~

~~$13.33 \angle -160^\circ$~~

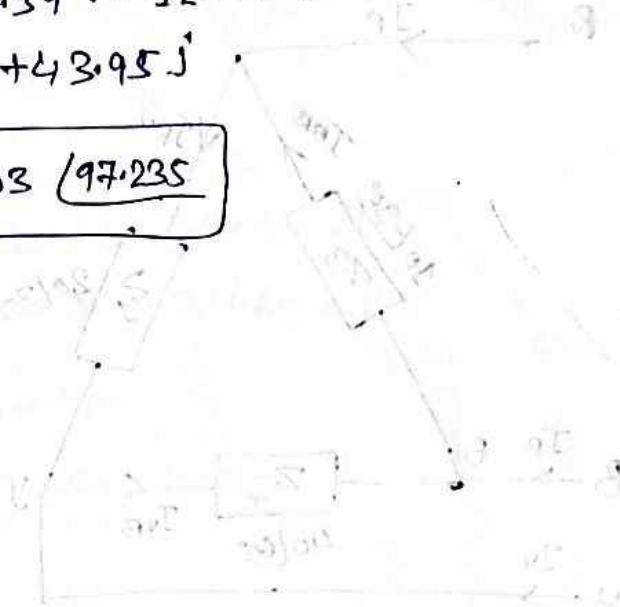
~~$I_B = 46.286 \angle 134.27^\circ$~~

$$I_B = 40 \angle 280^\circ - 13.33 \angle -160^\circ$$

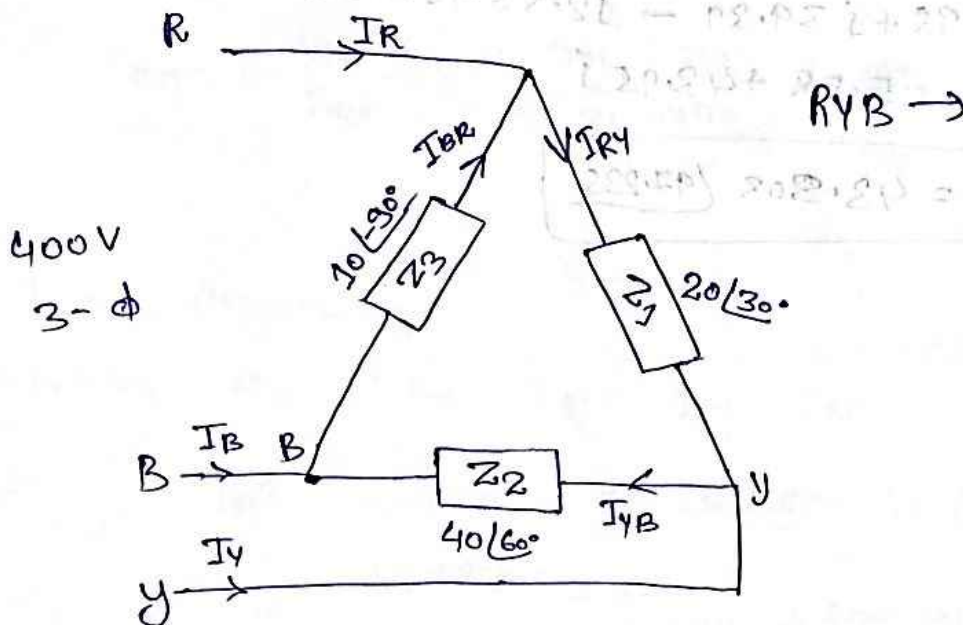
$$6.95 + j39.39 - 12.53 + j4.56$$

$$-5.58 + 43.95j$$

$$I_B = 43.803 \angle 97.235^\circ$$



\* Three impedances  $Z_1 = 20 \angle 30^\circ \Omega$ ,  $Z_2 = 40 \angle 60^\circ \Omega$  and  $Z_3 = 10 \angle -30^\circ \Omega$  are delta connected to a 400V 3- $\phi$  system. Determine (i) Phase currents (ii) Line currents (iii) Total power consumed by the load.



$$V_{RY} = 400 \angle 0^\circ$$

$$V_{YB} = 400 \angle -120^\circ$$

$$V_{BR} = 400 \angle -240^\circ$$

gn delta connected,

Line Voltages = Phase Voltages

$$V_{RN} = 400 \angle 0^\circ$$

$$V_{YN} = 400 \angle -120^\circ$$

$$V_{BN} = 400 \angle -240^\circ$$

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{400 \angle 0^\circ}{20 \angle 30^\circ} = \frac{400 \angle 0^\circ}{20 \angle 30^\circ} = 20 \angle -30^\circ$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{400 \angle -120^\circ}{40 \angle 60^\circ} = 10 \angle -180^\circ$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{400 \angle -240^\circ}{10 \angle -90^\circ} = 40 \angle -150^\circ$$

\* Line Current

$$I_R = I_{RV} - I_{BR}$$

$$I_R = 20 \angle -30^\circ - 40 \angle -150^\circ$$

$$I_R = 17.32 - j10 - (-34.64 - j20)$$

$$I_R = 17.32 - j10 + 34.64 + j20$$

$$= 51.96 + j10 =$$

$$I_R = 52.91 \angle 10.89^\circ$$

$$I_Y = I_{YV} - I_{YB} - I_{RY}$$

$$= 10 \angle -180^\circ - 20 \angle -30^\circ$$

$$= -10 + 0j - (17.32 - j10)$$

$$-10 - 17.32 + j10$$

~~$$-27.32 + j10 = 29.09 \angle 20.10^\circ$$~~

$$27.32 + j10 = I_Y = 29.09 \angle 20.10^\circ$$

$$I_B = I_{BR} - I_{YB}$$

$$= 40 \angle -150^\circ - 10 \angle -180^\circ$$

$$-34.64 - j20 - (-10 + 0j) = -34.64 - j20 + 10 =$$

$$-24.64 - j20$$

$$I_B = 31.74 \angle -140.33^\circ$$



(iii) Total power consumed by the load,  
for R phase

$$P_R = I_R^2 R_R$$

$$I_R^2 = I_{YB}^2$$

$R_R =$  from  $Z_1$

$$20 \angle 30^\circ = \frac{17.32 - j10}{R - jX_c}$$

$$\text{So } R = 17.32$$

~~$R_R =$~~   $I_R^2 = 20 \angle 30^\circ$  but for power, we take only magnitude,

$$P_R = (20)^2 \times 17.32 = 6928$$

$$P_Y = I_Y^2 R_Y$$

$$R_Y \text{ from } Z_2 = 40 \angle 60^\circ = \frac{20 + j34}{R + jX_L} \quad R = 20$$

$$P_Y = I_Y^2 = (I_{YB})^2 = 10 \angle -180^\circ = \text{we take only magnitude}$$

$$P_Y = (10)^2 \times 20 = 2000$$

$$P_B = I_B^2 R_B$$

$$R_B \text{ from } Z_3 = 10 \angle 90^\circ = \frac{0 - j10}{R - jX_c}$$

$$I_B^2 = I_{BR} = 40 \angle 150^\circ$$

$$P_B = 40 \cdot (0) = 0$$

$$\text{Total power} = P_R + P_Y + P_B$$

$$P_R = 6928 + 2000 + 0 = 8928 \text{ W}$$

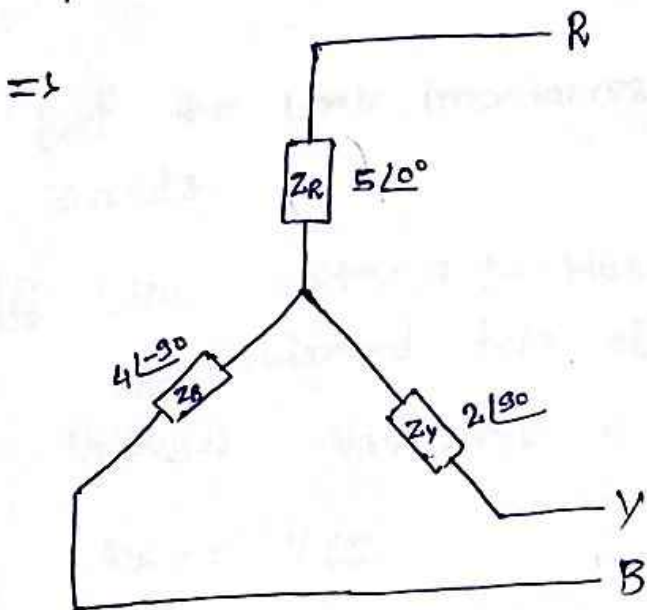
\* A Symmetrical 3-phase 100 volts 3 wire supply giving (fits) and unbalanced star connected load with impedances of the load as  $Z_R = 5 \angle 0^\circ \Omega$ ,  $Z_Y = 2 \angle 90^\circ \Omega$ ,  $Z_B = 4 \angle -90^\circ \Omega$

Find

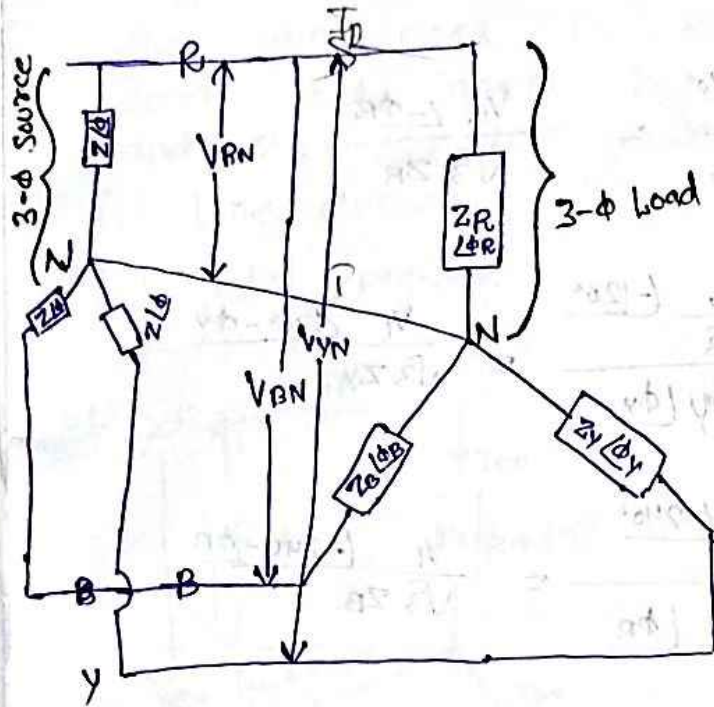
(i) Line Currents

(ii) Voltage across the impedances, using  $\Delta$  to

~~delta~~ delta conversion method.



# Analysis of unbalanced Star Connected Load :-



Let us consider an unbalanced star connected load which is supplied by a voltage

then the potential of source is equal to the potential of load

Hence the voltage of the source is equal to voltage of the load.

Let the phase impedances of load are  $Z_R \angle \phi_R$ ,  $Z_Y \angle \phi_Y$  and  $Z_B \angle \phi_B$

[i] Line Voltages :- Here the connected source is ~~very~~ balanced type of source then the line voltages  $V_{RY}$ ,  $V_{YB}$  and  $V_{BR}$  is given by,

$$V_{RY} = V \angle 0^\circ$$

$$V_{YB} = V \angle -120^\circ$$

$$V_{BR} = V \angle -240^\circ$$

[ii] phase Voltages :- The phase voltages  $V_{RN}$ ,  $V_{YN}$ , and  $V_{BN}$  are ~~can~~ given by

$$V_{RN} = \frac{V_L \angle 20^\circ}{\sqrt{3}}$$

$$V_{YN} = \frac{V_L \angle -120^\circ}{\sqrt{3}}$$

$$V_{BN} = \frac{V_L \angle -240^\circ}{\sqrt{3}}$$

UNIT - II

\* Phase Current:-  $I_{ph} = \frac{V_{ph}}{Z_p}$

$$I_R = \frac{V_{RN}}{Z_R} = \frac{V_L \angle 0^\circ}{\sqrt{3} Z_R \angle \phi_R} = \frac{V_L \angle -\phi_R}{\sqrt{3} Z_R}$$

$$I_Y = \frac{V_{YN}}{Z_Y} = \frac{V_L \angle -120^\circ}{\sqrt{3} Z_Y \angle \phi_Y} = \frac{V_L \angle -120^\circ - \phi_Y}{\sqrt{3} Z_Y}$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{V_L \angle -240^\circ}{\sqrt{3} Z_B \angle \phi_B} = \frac{V_L \angle -240^\circ - \phi_B}{\sqrt{3} Z_B}$$

In Star Connected System A phase current is equal to line current.

$$I_{ph} = I_{li}$$

Hence

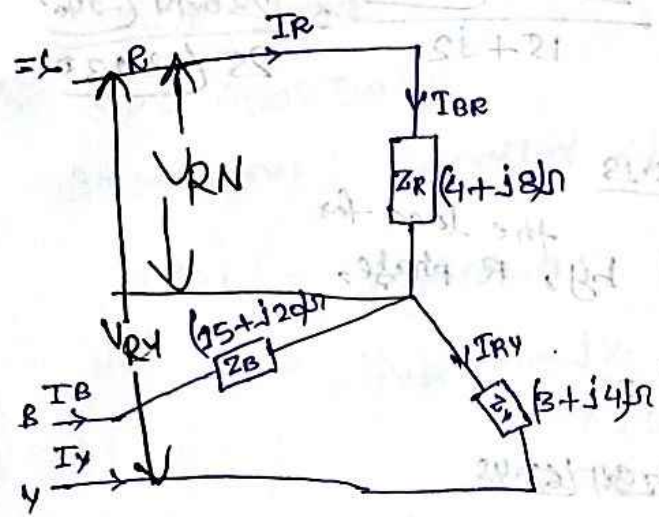
$$I_R = \frac{V_L \angle -\phi_R}{\sqrt{3} Z_R}$$

$$I_Y = \frac{V_L \angle -120^\circ - \phi_Y}{\sqrt{3} Z_Y}$$

$$I_B = \frac{V_L \angle -240^\circ - \phi_B}{\sqrt{3} Z_B}$$

\* An unbalanced star connected load has been supplied by a ~~balanced~~ voltage of 400V the impedances in each phase of the load are given by  $Z_R = 4 + j8 \Omega$ ,  $Z_Y = 3 + j4 \Omega$  and  $Z_B = 15 + j20 \Omega$  Calculate

- (i) Line current
- (ii) Total power.



Given data,

$Z_R = (4 + j8) \Omega$   
 $Z_Y = (3 + j4) \Omega$   
 $Z_B = (15 + j20) \Omega$

Supply Voltage = 400V

(i) Line ~~current~~ <sup>Voltage</sup>,

$V_{RY} = 400 \angle 0^\circ$   
 $V_{YB} = 400 \angle -120^\circ$   
 $V_{BR} = 400 \angle -240^\circ$



(ii) Phase Voltage

$V_{RN} = \frac{400 \angle 0^\circ}{\sqrt{3}}$   
 $V_{YN} = \frac{440 \angle -120^\circ}{\sqrt{3}}$  (9)  
 $V_{BN} = \frac{440 \angle -240^\circ}{\sqrt{3}}$



(iii) Phase Current,

$I_{Rn} = \frac{V_{ph}}{Z_{ph}} = \frac{V_{RN}}{4 + j8} = \frac{400 \angle 0^\circ}{\sqrt{3} (4 + j8)}$

$$I_Y = \frac{V_{YN}}{Z_Y} = \frac{440 \angle -120^\circ}{\frac{\sqrt{3}}{3+j4}} = \frac{440 \angle -120^\circ}{5 \angle 53.13^\circ}$$

$$I_Y = \frac{254.03 \angle -120^\circ}{5 \angle 53.13^\circ} = 50.806 \angle -173.13^\circ$$

$$I_B = \frac{V_{BR}}{Z_{ph}} = \frac{400 \angle -240^\circ}{\frac{230.94 \angle -240^\circ}{15+j20}} = \frac{230.94 \angle -240^\circ}{25 \angle 53.13^\circ} = 9.237 \angle -293.13^\circ$$

the load for

(ii) Total power consumed by 1 R phase,

$$(P_R) = I_R^2 R_R = I_R^2 R = 4$$

$$R_R \text{ from } Z_1 = 4 + j8 = \underline{4}$$

$$R + jX_L = R = 4$$

\* A delta connected 3- $\phi$  load has  $10\Omega$  between R and Y,  $6.36mH$  between Y and B and  $636\mu F$  between B and R. The supply voltage is  $400V$   $50Hz$ . Calculate the line currents for a RBY phase sequence.

$\Rightarrow$  Given data :-

3- $\phi$  load

$$S_v = 400V \ 50Hz$$

Impedances Connected between R and Y ( $Z_{RY}$ ) =  $10\Omega$

$$= 10 \angle 0^\circ$$

$$(Z_{YB}) = 6.36mH(\omega)$$

$$Z_{YB} = 0 + jX_L$$

$$= 0 + j2\pi fL$$

$$= j \times 2 \times \pi \times 50 \times 6.36 \times 10^{-3}$$

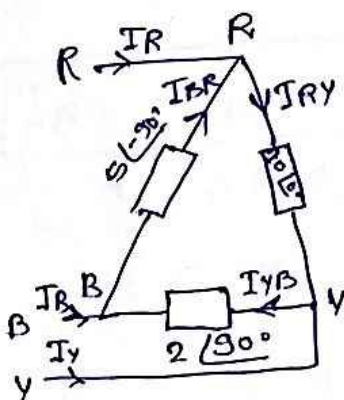
$$= j \ 1.99\Omega = 2 \angle 90^\circ$$

$$Z_{BR} = 636\mu F$$

$$= 0 - jX_C$$

$$= 0 - j \times \frac{1}{2\pi fC} = -j \times \frac{1}{2 \times \pi \times 50 \times 636 \times 10^{-6}}$$

$$Z_{BR} = -j5 = 5 \angle -90^\circ$$



Line Voltages (RBY) Sequence

$$V_{RY} = 400 \angle 0^\circ$$

$$V_{BR} = 400 \angle -120^\circ$$

$$V_{YB} = 400 \angle -240^\circ$$

Phase Voltages :- In  $\Delta$ -Connected

System  $V_L$  is  $V_{ph}$

$$\Rightarrow V_{ph} = V_{RY} = 400 \angle 0^\circ, V_{BR} = 400 \angle -120^\circ, V_{YB} = 400 \angle -240^\circ$$

(iii) Phase Current :-

$$I_{RY} = \frac{V_{RY}}{Z_{RY}} = \frac{400 \angle 0^\circ}{10 \angle 0^\circ} = 40 \angle 0^\circ = 40 \text{ A}$$

$$I_{YB} = \frac{V_{YB}}{Z_{YB}} = \frac{400 \angle -240^\circ}{2 \angle 90^\circ} = 200 \angle -330^\circ \text{ A}$$

$$I_{BR} = \frac{V_{BR}}{Z_{BR}} = \frac{400 \angle -120^\circ}{5 \angle -90^\circ} = 80 \angle -30^\circ \text{ A}$$

(iv) ~~Line Current :- In a delta connected system  $I_L = \sqrt{3} I_{ph}$~~

~~$I_R = 3 \times 40$~~

~~$I_Y = 200 \angle -330^\circ \times \sqrt{3}$~~

~~$I_B = 80 \angle -30^\circ \times \sqrt{3}$~~

(iv) Line Current :-  $I_R = I_{RY} - I_{BR}$

$$I_R = 40 - 80 \angle -30^\circ$$

$$40 - 69.28 + j40 =$$

$$-29.28 + j40 = 49.57 \angle 126^\circ$$

~~$I_Y$~~

$$I_Y = I_{YB} - I_{RY}$$

$$= 200 \angle -330^\circ - 40 \angle 0^\circ$$

$$173.20 + j100 - 40$$

$$133.2 + j100$$

$$= 166.56 \angle 36.89^\circ$$



$$I_B = I_{BR} - I_{YB}$$

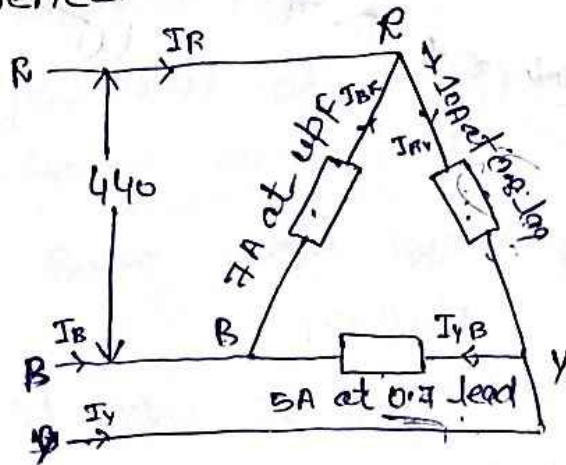
$$= 80 \angle -30^\circ - 200 \angle -330^\circ$$

$$= 69.28 - j40 - 173.20 - j100$$

$$= -103.92 - j140$$

$$I_B = 174.35 \angle -126.58^\circ$$

\* Find the line currents and power consumed by an unbalanced  $\Delta$ -delta connected load shown in fig. By assuming RYB phase sequence.



$$I_{ph} = 10 \text{ A}$$

$$\cos \theta = 0.8$$

$$\theta = \cos^{-1}(0.8)$$

$$I_{RY} = 10 \angle -36.86^\circ \text{ A}$$

$$I_{YB} = 5 \angle 45.57^\circ \text{ A}$$

$$I_{BR} = 7 \angle 0^\circ$$

$$I_R = I_{RY} - I_{BR}$$

$$= 10 \angle -36.86^\circ - 7 \angle 0^\circ$$

$$= 8.37 - j5.47 - 7$$

$$= 1.37 - j5.47 = 5.63 \angle -84.37^\circ \text{ Am}$$

$$I_Y = I_{YB} - I_{RY} = 5 \angle 45.57^\circ - 10 \angle -36.86^\circ$$

$$I_Y = 10.56 \angle 115.20$$

$$I_B = I_{BR} - I_{YB}$$

$$7 \angle 0^\circ - 5 \angle 45.57$$

$$= 7 + 0j - 3.500 - j3.570$$

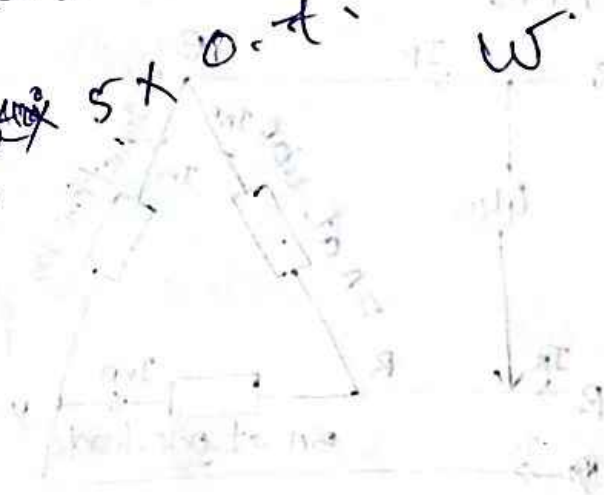
$$3.5 - j3.570 = 4.99 \angle 45.567$$

$$\text{power in R} = V \cdot I \cos \phi$$

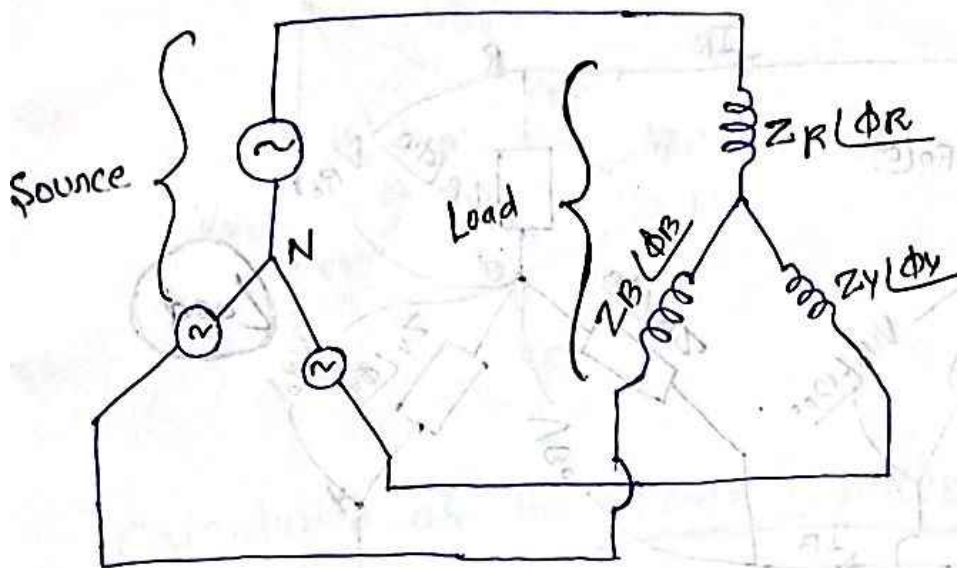
$$= \frac{440 \angle 10^\circ}{\sqrt{3}} \cdot 0.8$$

$$= 440 \cdot 5 \cdot 0.8$$

B.



# Unbalanced 3- $\phi$ , 3-wire System analysis! -



When suddenly neutral wire is broken, then potential of voltage are not equal in source and load.

→ To solve these system, there are three methods.

(i) Star to delta conversion

(ii) Milliman's method

(iii) Loop (or) Mesh method.

\* Milliman's method! - This is the method used for solving an unbalanced three wire star connected load.

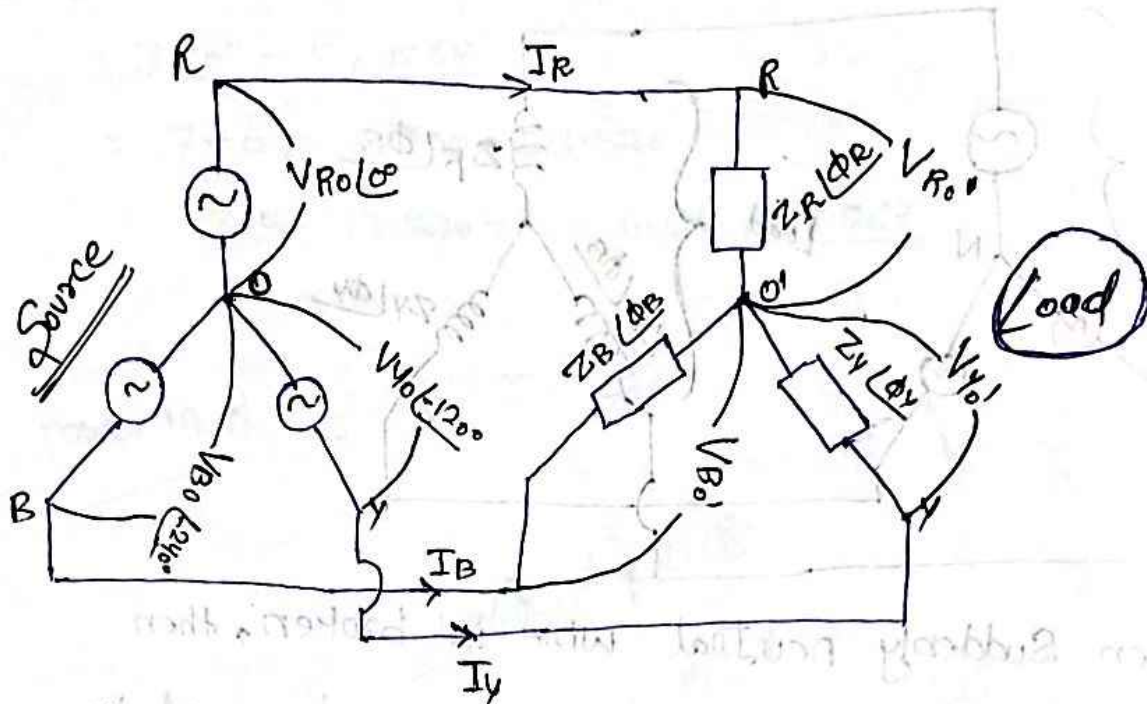
Let us consider an unbalanced star load is connected to balanced 3-phase supplying. Let  $V_{RO}$ ,  $V_{YO}$ ,  $V_{BO}$  are the phase voltages of

connected sources, Let  $V_{RO'}$ ,  $V_{YO'}$ ,  $V_{BO'}$  are

the phase voltages of the connected load.

The impedances of the load are  $Z_R \angle \phi_R$ ,

$Z_Y \angle \phi_Y$ ,  $Z_B \angle \phi_B$



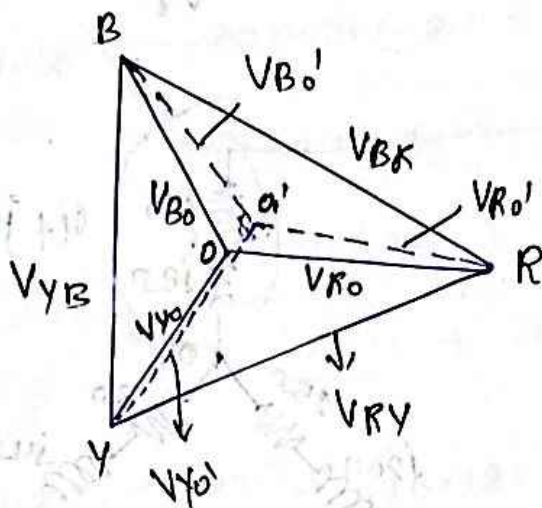
By considering triangular phasor diagram the line  $R_Y, Y_B, B_R$  represent line voltages by using millimans theorem.

The unknown voltage  $V_{00'}$  is calculated by using ~~the~~ the formula:

$$V_{00'} = \frac{V_{R0} Y_R + V_{Y0} Y_Y + V_{B0} Y_B}{Y_R + Y_Y + Y_B}$$

Where

$Y_R, Y_Y, Y_B$  } are admittances of the each branch.

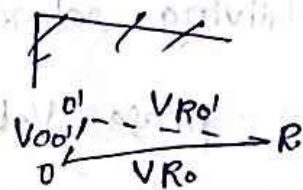


Calculation of the phase voltages from load

By considering  $\Delta RO'O$  then

$$V_{R0} = V_{O'O} + V_{R'O'}$$

$$V_{R'O'} = V_{R0} - V_{O'O}$$



$$V_{Y'O'} = V_{Y0} - V_{O'O}$$

$$V_{B'O'} = V_{B0} - V_{O'O}$$

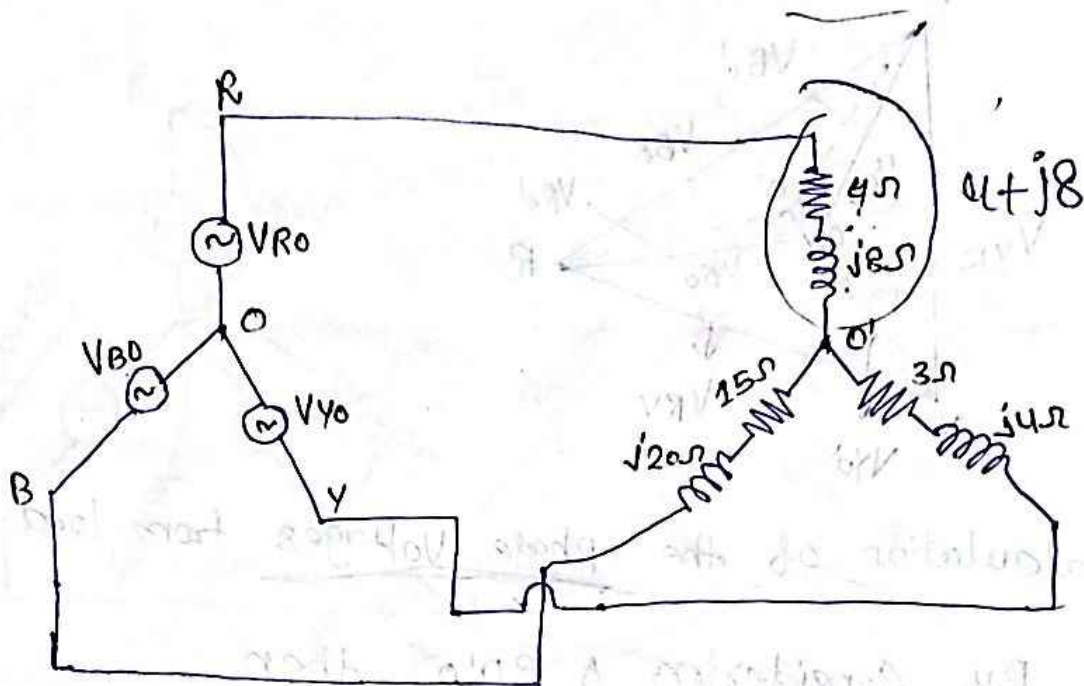
Phase current in 3- $\phi$ , 3-wire system,  
 $I_L = I_{ph}$

$$I_R = \frac{V_{R'O'}}{Z_R} = \frac{V_{R0} - V_{O'O}}{Z_R \cos \phi_R}$$

$$I_Y = \frac{V_{Y'O'}}{Z_Y} = \frac{V_{Y0} - V_{O'O}}{Z_Y \cos \phi_Y}$$

$$I_B = \frac{V_{B'O'}}{Z_B} = \frac{V_{B0} - V_{O'O}}{Z_B \cos \phi_B}$$

For the given circuit diagram calculate the 3 load phase voltages.  $V_{R0} = \frac{400 \angle -30^\circ}{\sqrt{3}}$



∴ Giving reference

phase voltage

$$V_{R0} = \frac{400 \angle -30^\circ}{\sqrt{3}} = 230.94 \angle -30^\circ$$

$$V_{Y0} = 230.94 \angle (-30^\circ - 120^\circ) = 230.94 \angle -150^\circ$$

$$V_{B0} = 230.94 \angle (-30^\circ - 240^\circ) = 230.94 \angle -270^\circ$$

$$V_{O'0} = \frac{V_{R0} Y_R + V_{Y0} Y_Y + V_{B0} Y_B}{Y_R + Y_Y + Y_B}$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{4 + j8} = 0.11 \angle -63.43^\circ \text{ } \Omega$$

$$Y_Y = \frac{1}{3 + j4} = 0.2 \angle -53.13^\circ \text{ } \Omega$$

$$Y_B = \frac{1}{15 + j20} = 0.04 \angle -53.1^\circ \text{ } \Omega$$

Now

$$V_{010} = (230.94 \angle -30^\circ) \cdot (0.41 \angle -63.43^\circ) +$$

$$(230.94 \angle -150^\circ) \cdot (0.2 \angle -53.13^\circ) +$$

$$(230.94 \angle -270^\circ) \cdot (0.04 \angle -53.1^\circ)$$

$$\frac{0.11 \angle -63.43^\circ + 0.2 \angle -53.13^\circ + 0.04 \angle -53.1^\circ}{}$$

$$25.40 \angle -93.43^\circ + 46.188 \angle -96.87^\circ + 9.2376 \angle -216.9^\circ$$

$$0.05 - j0.098 + 0.12 - j0.159 + 0.024 - j0.031$$

$$-1.519 - j25.35 - 5.52 - j45.86 - 7.387 + j5.546$$

$$0.194 - j0.288$$

$$\frac{-14.426 - j65.664}{0.194 - j0.288}$$

$$\frac{67.229 \angle -77.609^\circ}{0.347 \angle -56.035^\circ}$$

$$V_{010} = 193.74 \angle -21.574^\circ$$

$$\alpha_1 \angle \phi_1 \times \alpha_2 \angle \phi_2$$

$$\alpha_1 \angle \phi_1 \quad \alpha_2 \angle -\phi_2$$

$$\Rightarrow \alpha_1 \alpha_2 \angle \phi_1 + \phi_2$$

$$\alpha_1 \alpha_2 \angle \phi_1 - \phi_2$$

$$\alpha_1 \angle -\phi_1 \quad \alpha_2 \angle -\phi_2$$

$$\alpha_1 \alpha_2 \angle -\phi_1 - \phi_2$$

$$\frac{\alpha_1 \angle \phi_1}{\alpha_2 \angle -\phi_2}$$

$$\left(\frac{\alpha_1}{\alpha_2}\right) \angle -\phi_1 + \phi_2$$

$$\text{Now } V_{00}' = \frac{V_{R0}Y_R + V_{Y0}Y_Y + V_{B0}Y_B}{Y_R + Y_B + Y_Y} =$$

$$\frac{[(230.94 \angle -30^\circ)(0.11 \angle -63.43^\circ)] + [(230.94 \angle -150^\circ)(0.2 \angle -53.13^\circ)] + [(230.94 \angle -270^\circ)(0.04 \angle -53.1^\circ)]}{0.11 \angle -63.43^\circ + 0.2 \angle -53.13^\circ + 0.04 \angle -53.1^\circ} =$$

$$= \frac{25.4034 \angle -93.43^\circ + 46.188 \angle -203.13^\circ + 9.2376 \angle -323.1^\circ}{0.11 \angle -63.43^\circ + 0.2 \angle -53.13^\circ + 0.04 \angle -53.1^\circ}$$

$$= \frac{-1.51 - j25.35 - 42.47 + j18.14 + 7.38 + j5.54}{0.04 - j0.09 + 0.12 - j0.15 + 0.02 - j0.03}$$

$$= \frac{-36.6 - j1.67}{0.18 - j0.27} = \frac{36.63 \angle -177.38^\circ}{0.32 \angle -56.30^\circ} =$$

$$V_{00}' = 104.8 \angle -120.6^\circ$$

$$V_{00}' = 104.8 \angle -120.6^\circ$$

$$V_{R0}' = V_{R0} - V_{00}'$$

$$= 230.94 \angle -30^\circ - 104.8 \angle -120.6^\circ =$$

$$= 199.99 - j115.49 + 53.347 + j90.205$$

$$= 253.337 - j25.265$$

$$V_{R0}' = 254.59 \angle -5.895^\circ$$

$$V_{Y0}' = V_{Y0} - V_{00}'$$

$$230.94 \angle -150^\circ - 104.8 \angle -120.6^\circ$$

$$126.14 \angle -270.6^\circ$$

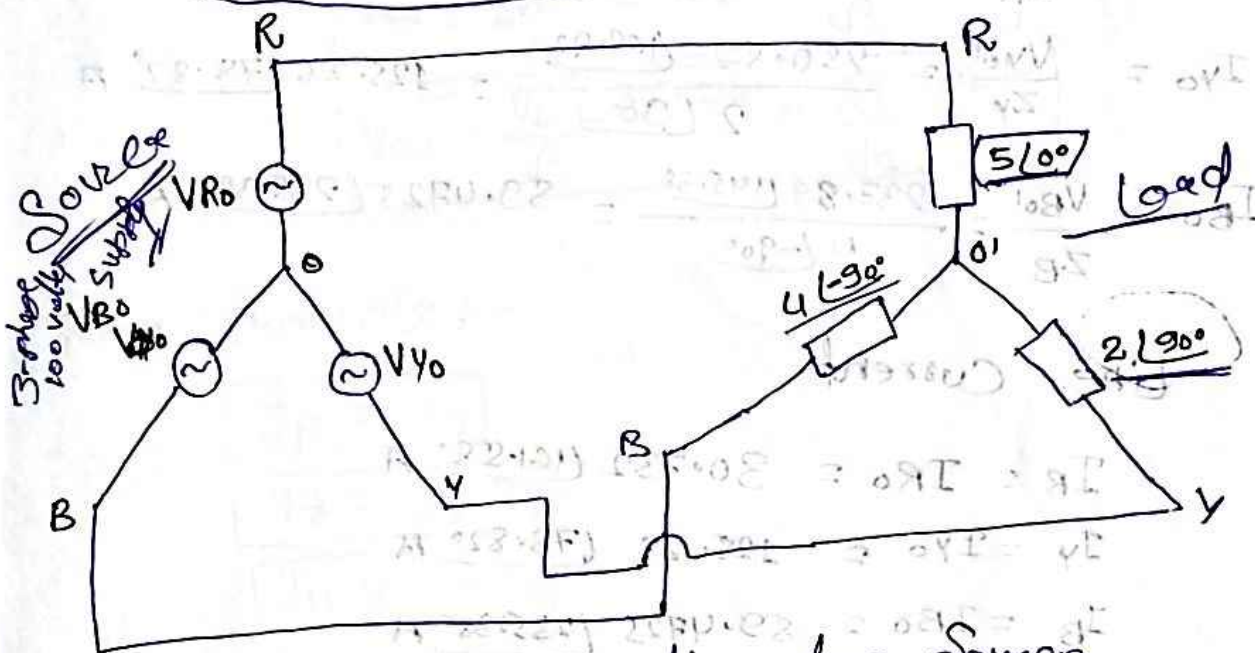
$$= \begin{matrix} -199.99 - j115.49 + 53.34 + j90.20 \\ -146.65 - j25.27 \end{matrix}$$



$$V_{y0} = 148.81 \angle -170.22$$

\* A Symmetrical 3-phase 100 volts 3-wire supply feed and unbalanced star connection load with impedances of the load as  $Z_R = 5 \angle 0^\circ$ ,  $Z_Y = 2 \angle 90^\circ$ ,  $Z_B = 4 \angle -90^\circ$  find the line currents using millimane method.

Note :-  $V_{R0}$  is  $\frac{100 \angle -30^\circ}{\sqrt{3}}$



Giving Reference:- phase voltage from source

phase voltage:  $V_{R0} = \frac{100 \angle 30^\circ}{\sqrt{3}} = 57.73 \angle -30^\circ$

$$V_{Y0} = \frac{100 \angle -150^\circ}{\sqrt{3}} = 57.73 \angle -150^\circ$$

$$V_{B0} = \frac{100 \angle -270^\circ}{\sqrt{3}} = 57.73 \angle -270^\circ$$

$$V_{o0} = \frac{V_{R0} Y_R + V_{Y0} Y_Y + V_{B0} Y_B}{Y_R + Y_Y + Y_B}$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{5 \angle 0^\circ} = 0.2 \angle 0^\circ$$

$$Y_Y = \frac{1}{2 \angle 90^\circ} = 0.5 \angle -90^\circ$$

$$Y_B = \frac{1}{4 \angle -90^\circ} = 0.25 \angle 90^\circ$$

Where  $Y_R$ ,  $Y_Y$ ,  $Y_B$  } Admittance of each branch

$$V_{00'} = \frac{(57.73 \angle -30^\circ)(0.2 \angle 0^\circ) + (57.73 \angle -150^\circ)(0.5 \angle 90^\circ) + (57.73 \angle -270^\circ)(0.25 \angle 90^\circ)}{0.2 \angle 0^\circ + 0.5 \angle -90^\circ + 0.25 \angle 90^\circ}$$

$$V_{00'} = \frac{11.546 \angle -30^\circ + 28.865 \angle -240^\circ + 14.4325 \angle -180^\circ}{0.2 \angle 0^\circ + 0.5 \angle -90^\circ + 0.25 \angle 90^\circ} = \frac{9.99 - j5.77 - 14.28 + j24.73 - 14.4325}{0.2 - j0.5 + j0.25}$$

$$= \frac{-18.7225 - 161.04j}{0.2 - 0.75j} = \frac{162.12 \angle -96.63^\circ}{0.77 \angle -75.06^\circ} = \boxed{210.54 \angle -21.57^\circ}$$

Phase Voltage from load,

$$V_{R0'} = V_{R0} - V_{00'} ; V_{R0'} = 57.73 \angle -30^\circ - 210.54 \angle -21.57^\circ = 153.66 \angle 161.58^\circ$$

$$V_{Y0'} = V_{Y0} - V_{00'} ; 57.73 \angle -150^\circ - 210.54 \angle -21.57^\circ = 250.52 \angle 168.82^\circ$$

$$V_{B0'} = V_{B0} - V_{00'} ; 57.73 \angle -270^\circ - 210.54 \angle -21.57^\circ = 237.89 \angle 145.38^\circ$$

Phase Current

$$I_{R0} = \frac{V_{R0'}}{Z_R} = \frac{153.66 \angle 161.58^\circ}{5 \angle 0^\circ} = 30.732 \angle 161.58^\circ \text{ A}$$

$$I_{Y0} = \frac{V_{Y0'}}{Z_Y} = \frac{250.52 \angle 168.82^\circ}{2 \angle 90^\circ} = 125.26 \angle 78.82^\circ \text{ A}$$

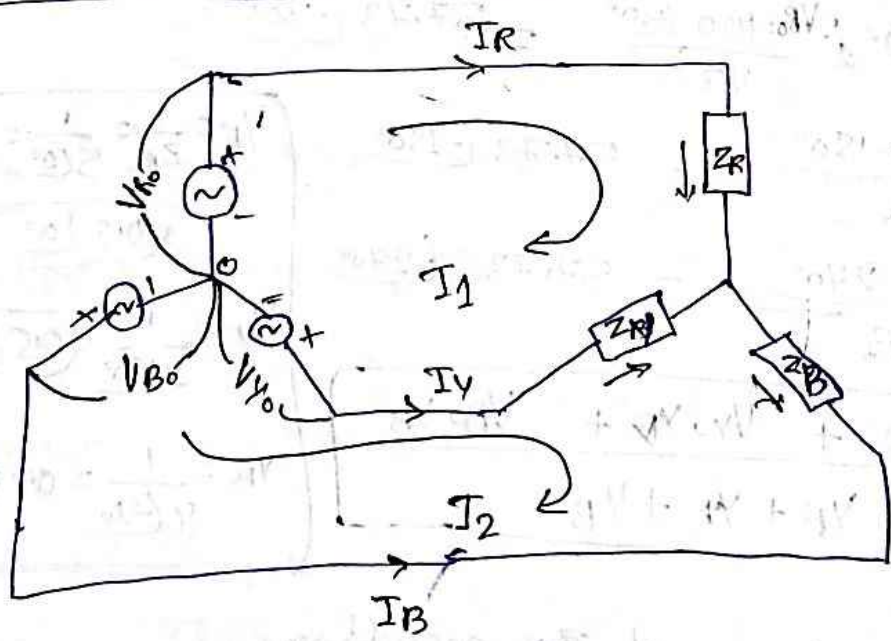
$$I_{B0} = \frac{V_{B0'}}{Z_B} = \frac{237.89 \angle 145.38^\circ}{4 \angle -90^\circ} = 59.4725 \angle 235.38^\circ \text{ A}$$

Line Current

$$I_R = I_{R0} = 30.732 \angle 161.58^\circ \text{ A}$$

$$I_Y = I_{Y0} = 125.26 \angle 78.82^\circ \text{ A}$$

$$I_B = I_{B0} = 59.4725 \angle 235.38^\circ \text{ A}$$



## Loop (or) Mesh Method:-

Let us consider an unbalanced 3-phase 3-wire star connected load which is connected to a 3-phase balanced source.

Let assume RYB phase sequence, then the source phase voltage is given by.

$$V_{R0} = \frac{V_L \angle -30^\circ}{\sqrt{3}} = \frac{V_L \angle -30^\circ}{\sqrt{3}}$$

$$V_{Y0} = \frac{V_L \angle -30^\circ - 120^\circ}{\sqrt{3}} = \frac{V_L \angle -150^\circ}{\sqrt{3}}$$

$$V_{B0} = \frac{V_L \angle -30^\circ - 240^\circ}{\sqrt{3}} = \frac{V_L \angle -270^\circ}{\sqrt{3}}$$

Phase currents:-

$$I_R = I_1$$

$$I_Y = I_1 - I_2$$

$$I_B = -I_2$$

By applying KVL for loop (1)

$$I_1 Z_R + Z_Y (I_1 - I_2) + V_{Y0} = V_{R0}$$

$$I_1 (Z_R + Z_Y) - I_2 Z_Y = V_{R0} - V_{Y0} \quad \text{--- (i)}$$

By applying KVL for loop (2)

$$Z_Y (I_2 - I_1) + Z_B (I_2) + V_{B0} = V_{Y0}$$

$$-Z_Y I_1 + (Z_B + Z_Y) I_2 = V_{Y0} - V_{B0} \quad \text{--- (ii)}$$

From eqn (i) & (ii) in matrix form;

$$\begin{bmatrix} (Z_R + Z_Y) & -Z_Y \\ -Z_Y & (Z_B + Z_Y) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_{R0} - V_{Y0} \\ V_{Y0} - V_{B0} \end{bmatrix}$$

By applying Cramer's Rule :-

$$I_1 = \frac{\begin{vmatrix} (V_{R0} - V_{Y0}) & -Z_Y \\ (V_{Y0} - V_{B0}) & (Z_B + Z_Y) \end{vmatrix}}{\begin{vmatrix} Z_R + Z_Y & -Z_Y \\ -Z_Y & (Z_B + Z_Y) \end{vmatrix}}$$

$$I_2 = \frac{\begin{vmatrix} (Z_R + Z_Y) & V_{R0} - V_{Y0} \\ -Z_Y & V_{Y0} - V_{B0} \end{vmatrix}}{\begin{vmatrix} Z_R + Z_Y & -Z_Y \\ -Z_Y & Z_B + Z_Y \end{vmatrix}}$$

Line Current:- In star connected system (Load) system line current is equal to phase current.

$$I_L = I_{ph}$$

then,

$$I_R = I_1$$

$$I_Y = I_1 - I_2$$

$$I_B = -I_2$$

\* phase Voltages:- The phase voltage across R branch is

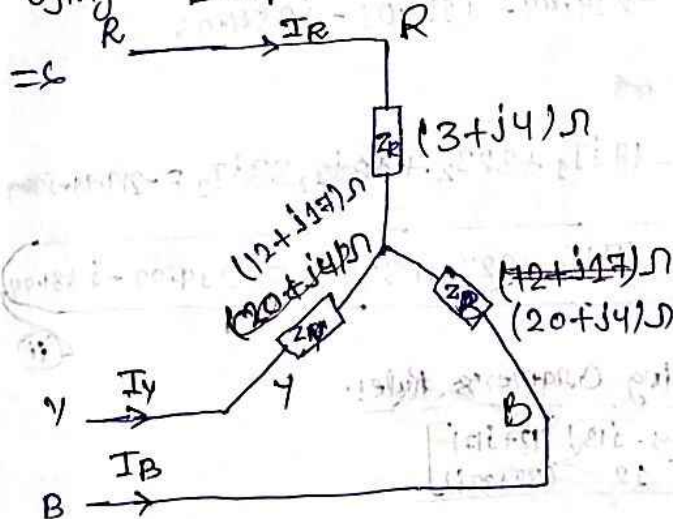
$$V_R = I_R Z_R$$

$$V_Y = I_Y Z_Y$$

$$V_B = I_B Z_B$$

\* A 3-phase 3-Wire Star Connected load has impedances in each phase  $(3+j4)\Omega$ ,  $(12+j17)\Omega$ , and  $(20+j14)\Omega$  are connected in each phase with 440V 50HZ supply calculate the load phase voltages  $[V_R, V_Y, V_B]$

using Loop methods.



Given data:-

phase voltage = 440V

V<sub>ph</sub>

$$2 \frac{1}{\sqrt{3}}$$

$$V_{R0} = \frac{440}{\sqrt{3}} \angle -30^\circ = 254.03 \angle -30^\circ$$

$$V_{Y0} = \frac{440}{\sqrt{3}} \angle -150^\circ = 254.03 \angle -150^\circ$$

$$V_{B0} = \frac{440}{\sqrt{3}} \angle -270^\circ = 254.03 \angle -270^\circ$$

By applying KVL for Loop (i)

$$I_1(3+j4) + (12+j17)(I_2-I_1) =$$

$$254.03 \angle -30^\circ - 254.03 \angle -150^\circ$$

$$I_1(3+j4) + (12+j17)I_2 - (12+j17)I_1 =$$

$$219.99 - j127.015 - 219.99 + j127.015$$

$$(3+j4)I_1 + (12+j17)I_2 -$$

$$3I_1 + (j4)I_1 + 12I_2 + (j17)I_2 - 12I_1 - (j17)I_2 =$$

$$0 - j5 \times 10^3$$

$$-9I_1 + j13I_1 + 12I_2 + (j17)I_2 = -5j \times 10^3$$

Applying KVL for Loop (ii)

$$-(12+j17)I_1 + [20+j4+12+j17]I_2 =$$

$$254.03 \angle -150^\circ - 254.03 \angle -270^\circ$$

$$\Rightarrow -12I_1 - j17I_1 + 20I_2 + 4jI_2 + 12I_2 + 17jI_2 =$$

$$-219.99 - j127.01 - j254.03$$

$$= 0$$

$$12I_1 - 17jI_1 + 32I_2 + 33jI_2 = -219.99 - j381.04$$

$$12I_1 - 17jI_1 + 32I_2 + 33jI_2 = -219.99 - j381.04$$

Applying Cramer's Rule:-

$$\begin{vmatrix} (-9+j13) & (12+j17) \\ 12 & (32+33j) \end{vmatrix}$$

\* Applying Cramer's Rule:-  
~~mmmmmm~~ ~~mmmmmm~~ ~~mmmm~~

from eq (ii) & (iii) in matrix form

$$\begin{bmatrix} -9 - j13 & 12 + j17 \\ 12 & 15 + j33 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -5j \times 10^{-3} \\ -219.99 - j381.04 \end{bmatrix}$$

By applying Cramer's Rule:-

$$I_1 = \frac{\begin{vmatrix} -5j \times 10^{-3} & 12 + j17 \\ -219.99 - j381.04 & 15 + j33 \end{vmatrix}}{\begin{vmatrix} -9 - j13 & 12 + j17 \\ 12 & 15 + j33 \end{vmatrix}} =$$

$$I_1 = \frac{(-5j \times 10^{-3})(15 + j33) - (12 + j17)(-219.99 - j381.04)}{(-9 - j13)(15 + j33) - (12)(12 + j17)}$$

$$I_1 = \frac{(5 \times 10^{-3} \angle -90^\circ)(36.24 \angle 65.55^\circ) - [20.80 \angle 54.78^\circ](429.98 \angle -119.99^\circ)}{(15.81 \angle -124.69^\circ)(36.24 \angle 65.55^\circ) - 12 \angle 0^\circ (20.80 \angle 54.78^\circ)}$$

$$= \frac{181200 \angle -24.45^\circ - 9151.584 \angle -65.21^\circ}{572.95 \angle 59.14^\circ - 249.6 \angle -54.78^\circ}$$

$$= \frac{164950.49 - j74 - 3837.20 + j8308.127}{293.89 - j492.83 - 143.94 + j203.90}$$

$$= \frac{161113.29 + j8234.27j}{149.95 - j287.93} = \frac{161323.57 \angle 2.92^\circ}{324.638 \angle -62.49^\circ}$$

$$= 496.94 \angle 65.41^\circ$$

\* A star connected unbalanced system has the impedances in each branch are given by  $Z_R = 6 \angle 40^\circ \Omega$ ,  $Z_Y = 8 \angle 30^\circ \Omega$ ,  $Z_B = 3 \angle 0^\circ \Omega$  is connected to 400V 3-phase supply. Calculate the line currents by using

(i) Loop Methods and (ii) Millman's method.

∴ From Loop method,

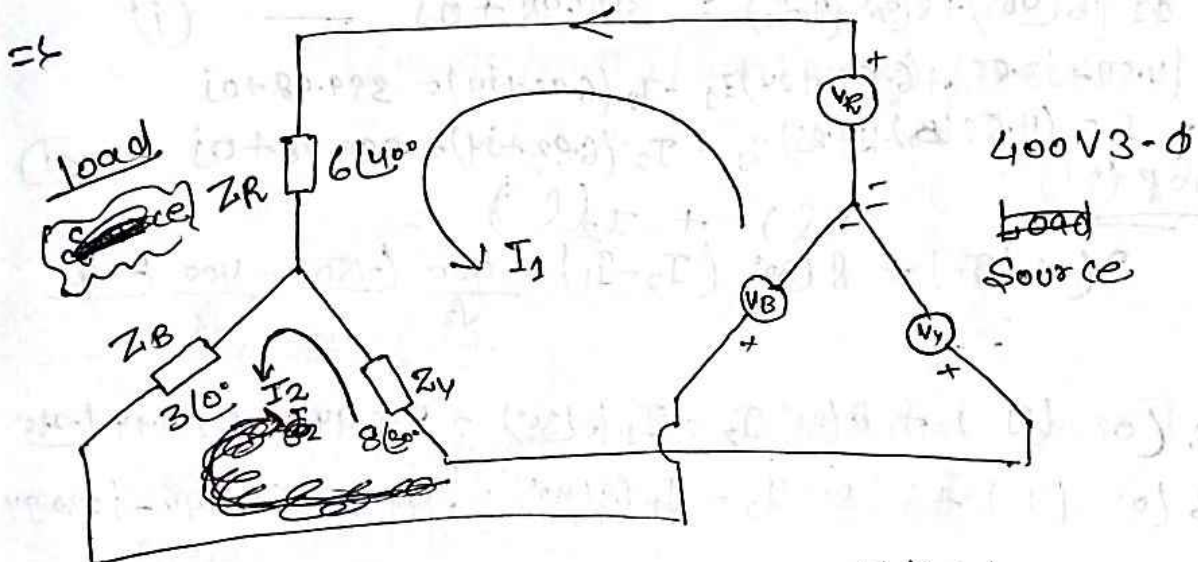
Given data.

$$Z_R = 6 \angle 40^\circ \Omega$$

$$Z_Y = 8 \angle 30^\circ \Omega$$

$$Z_B = 3 \angle 0^\circ \Omega$$

3-phase 400V



Line Voltages

$$V_{RY} = 400 \angle -30^\circ$$

$$V_{YB} = 400 \angle -30^\circ - 120^\circ$$

$$V_{BR} = 400 \angle -30^\circ - 240^\circ$$

Phase Voltages

$$V_R = \frac{400 \angle -30^\circ}{\sqrt{3}}$$

$$V_Y = \frac{400 \angle -150^\circ}{\sqrt{3}}$$

$$V_B = \frac{400 \angle -270^\circ}{\sqrt{3}}$$

9n Loop 1:-

$$I_1 (Z_R + Z_Y) - I_2 (Z_Y) + V_Y = V_R$$

$$I_1 (Z_R + Z_Y) - I_2 (Z_Y) = V_R - V_Y$$



loop (1)

$$I_1 (6 \angle 40^\circ) + 8 \angle 30^\circ (I_1 - I_2)$$

$$= \frac{400}{\sqrt{3}} \angle -30^\circ - \frac{400}{\sqrt{3}} \angle -150^\circ$$

$\uparrow$   $I_1$

$$I_1 (6 \angle 40^\circ) + 8 \angle 30^\circ (I_1 - I_2) = 230.94 \angle -30^\circ - 230.94 \angle -150^\circ$$

$$I_1 (6 \angle 40^\circ) + 8 \angle 30^\circ (I_1 - I_2) = 199.99 - j115.47 + 199.99 + j115.47$$

$$I_1 (6 \angle 40^\circ) + 8 \angle 30^\circ (I_1 - I_2) = 399.98 + 0j \quad \text{--- (i)}$$

$$I_1 (4.59 + j3.85) + (6.92 + j4)I_1 - I_2 (6.92 + j4) = 399.98 + 0j$$

$$= I_1 (11.51 + j7.85) - I_2 (6.92 + j4) = 399.98 + 0j \quad \text{--- (ii)}$$

loop (2)

$$3 \angle 0^\circ (I_2) + 8 \angle 30^\circ (I_2 - I_1) = \frac{400}{\sqrt{3}} \angle -150^\circ - \frac{400}{\sqrt{3}} \angle -270^\circ$$

$$3 \angle 0^\circ (I_2) + 8 \angle 30^\circ I_2 - I_1 (8 \angle 30^\circ) = 230.94 \angle -150^\circ - 230.94 \angle -270^\circ$$

$$3 \angle 0^\circ (I_2) + 8 \angle 30^\circ I_2 - I_1 (8 \angle 30^\circ) = -199.99 - j115.47 - j230.94$$

$$3 + 0j (I_2) + (6.92 + j4)I_2 - (6.92 + j4)I_1 = -199.99 - j346.38$$

$$(9.92 + j4)I_2 - (6.92 + j4)I_1 = -199.99 - j346.38 \quad \text{--- (iii)}$$

eqn (ii) & (iii) in matrix form,

$$\begin{bmatrix} 11.51 + j7.85 & -6.92 - j4 \\ -6.92 - j4 & 9.92 + j4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 399.98 + 0j \\ -199.99 - j346.38 \end{bmatrix}$$

By Cramer's Rule

$$I_1 = \frac{\begin{vmatrix} 399.98 + 0j & -6.92 - j4 \\ -199.99 - j346.38 & 9.92 + j4 \end{vmatrix}}{\begin{vmatrix} 11.51 + j7.85 & -6.92 - j4 \\ -6.92 - j4 & 9.92 + j4 \end{vmatrix}}$$

$$\Rightarrow \frac{399.98 \cdot I_1 \left[ (399.98 \angle 0^\circ) (10.69 \angle 21.96^\circ) \right] - \left[ (399.98 \angle 0^\circ) (-7.99 \angle 30.02^\circ) \right]}{\left[ (13.93 \angle 34.29^\circ) (10.69 \angle 21.96^\circ) \right] - \left[ (-7.99 \angle 30.02^\circ) (-7.99 \angle 30.02^\circ) \right]}$$

$$I_1 = \frac{4275.78 \angle 21.96^\circ}{148.91 \angle 56.25^\circ} - \frac{3195.68 \angle 90.01^\circ}{63.8401 \angle 60.04^\circ}$$

$$I_1 = \frac{399.98 \angle 0^\circ (10.69 \angle 21.96^\circ) - \left[ -\sqrt{(6.92 + j4)(199.99 + j346.38)} \right]}{\left[ (13.93 \angle 34.29^\circ) (10.69 \angle 21.96^\circ) \right] - \left[ -\sqrt{(6.92 + j4)(6.92 + j4)} \right]}$$

$$I_1 = \frac{4275.78 \angle 21.96^\circ - \left[ -\sqrt{(7.99 \angle 30.02^\circ)(399.98 \angle 0^\circ)(59.99)} \right]}{148.91 \angle 56.25^\circ - \left[ -\sqrt{(7.99 \angle 30.02^\circ)(7.99 \angle 30.02^\circ)} \right]}$$

$$I_1 = \frac{4275.78 \angle 21.96^\circ - \left[ -\sqrt{(3195.68 \angle 90.01^\circ)} \right]}{148.91 \angle 56.25^\circ - \left[ -\sqrt{(63.8401 \angle 60.04^\circ)} \right]}$$

$$I_1 = \frac{3965.55 + j1598.96 - 0.55 + j3195.67}{148.91 \angle 56.25^\circ} = \frac{82.72 + j123.81 + 31.88 + 55.30j}{114.6 + j179.11j}$$

$$I_1 = \frac{3965 + j4794.63}{114.6 + j179.11j} = \frac{6221.71 \angle 50.41^\circ}{212.63 \angle 57.38^\circ}$$

By Cramer's Rule

$$I_2 = \frac{\begin{vmatrix} 11.51 + j7.85 & 399.98 + 0j \\ -(6.92 + j4) & -(199.99 + j346.38) \end{vmatrix}}{\begin{vmatrix} 11.51 + j7.85 & -(6.92 + j4) \\ -(6.92 + j4) & 9.92 + j4 \end{vmatrix}}$$

$$\begin{vmatrix} 11.51 + j7.85 & -(6.92 + j4) \\ -(6.92 + j4) & 9.92 + j4 \end{vmatrix}$$

from the calculation part of  $I_2$

$$I_2 = \frac{(13.93 \angle 34.29^\circ)(-399.99 \angle 59.99^\circ) - (399.98 \angle 0^\circ)(-7.99 \angle 30.02^\circ)}{114.6 + 179.11j}$$

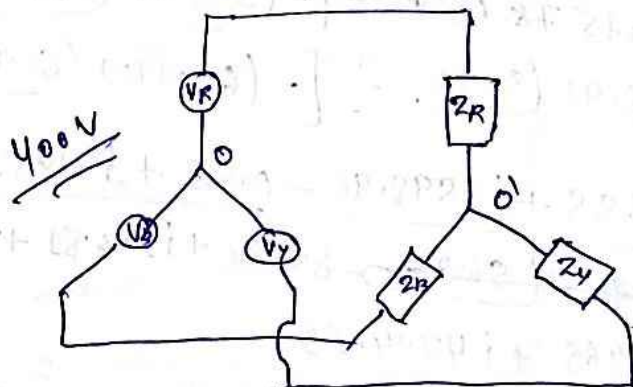
$$I_2 = \frac{(-5571.44 \angle 94.28^\circ) + (3195.84 \angle 30.02^\circ)}{114.6 + 179.11j}$$

$$I_2 = \frac{415.80 - j5559.90 + 3195.98 \angle 2767.12^\circ + j1598.88}{114.6 + 179.11j}$$

$$I_2 = \frac{3182.92 - j3959.02}{114.6 + 179.11j} = \frac{5078.28 \angle -51.18^\circ}{212.63 \angle 57.38^\circ}$$

$$I_2 = 23.88 \angle -108.56^\circ$$

By Millman's method



Phase Voltage:

$$V_{R0} = \frac{400}{\sqrt{3}} \angle -30^\circ = 230.94 \angle -30^\circ$$

$$V_{Y0} = \frac{400}{\sqrt{3}} \angle -150^\circ = 230.94 \angle -150^\circ$$

$$V_{B0} = 230.94 \angle -270^\circ$$

$$V_{00'} = \frac{V_{R0} Y_R + V_{B0} Y_B + V_{Y0} Y_Y}{Y_R + Y_B + Y_Y}$$

$$Y_R = \frac{1}{Z_R} = \frac{1}{6 \angle 40^\circ} = 0.16 \angle -40^\circ$$

$$Y_Y = \frac{1}{Z_Y} = \frac{1}{8 \angle 30^\circ} = 0.125 \angle -30^\circ$$

$$Y_B = \frac{1}{Z_B} = \frac{1}{3 \angle 0^\circ} = 0.33 \angle 0^\circ$$

$$V_{00'} = \frac{(230.94 \angle -30^\circ)(0.16 \angle -40^\circ) + (230.94 \angle -270^\circ)(0.33 \angle 0^\circ) + (230.94 \angle -150^\circ)(0.125 \angle -30^\circ)}{0.16 \angle -40^\circ + 0.125 \angle -30^\circ + 0.33 \angle 0^\circ}$$

$$V_{00'} = \frac{36.95 \angle 70^\circ + 76.21 \angle -270^\circ + 28.86 \angle -180^\circ}{0.12 - j0.20 + 0.10 - j0.06 + 0.33 + 0j}$$

$$V_{00'} = \frac{72.63 - j34.72 + 0 + j76.21 - 28.86 + 0j}{0.55 - j0.16}$$

$$V_{00'} = \frac{-16.23 + j41.49}{0.55 - j0.16} = \frac{44.55 \angle 111.36^\circ}{0.57 \angle -16.22^\circ}$$

$$V_{00'} = 78.15 \angle 127.58^\circ$$

Phase voltage from load

$$V_{R0'} = V_{R0} - V_{00'} = 230.94 \angle -30^\circ - 78.15 \angle 127.58^\circ =$$

$$V_{R0'} = 199.99 - j115.47 + 47.66 - j62.93 =$$

$$V_{R0'} = 247.65 - j177.4$$

$$V_{Y0}' = V_{Y0} - V_{00}' = -199.99 - j115.47 + 47.66 - j61.93$$

$$V_{Y0}' = -152.33 - j177.4$$

$$V_{B0}' = V_{B0} - V_{00}' = 0 + j230.94 + 47.66 - j61.93$$

$$V_{B0}' = 47.66 + j169.01$$

Phase Current

$$I_{R0} = \frac{V_{R0}'}{Z_R} = \frac{304.63 \angle -35.61^\circ}{6 \angle 40^\circ} = 50.77 \angle -75.62^\circ$$

$$I_{Y0} = \frac{V_{Y0}'}{Z_Y} = \frac{233.82 \angle -130.65^\circ}{8 \angle 30^\circ} = 29.22 \angle -180.65^\circ$$

$$I_{B0} = \frac{V_{B0}'}{Z_B} = \frac{175.60 \angle 74.25^\circ}{3 \angle 0^\circ} = 58.53 \angle 74.25^\circ$$

Line Current

$$I_R = I_{R0} = 50.77 \angle -75.62^\circ$$

$$I_Y = I_{Y0} = 29.22 \angle -180.65^\circ$$

$$I_B = I_{B0} = 58.53 \angle 74.25^\circ$$

By loop method

$$I_R = I_1 = 29.26 \angle -6.99^\circ$$

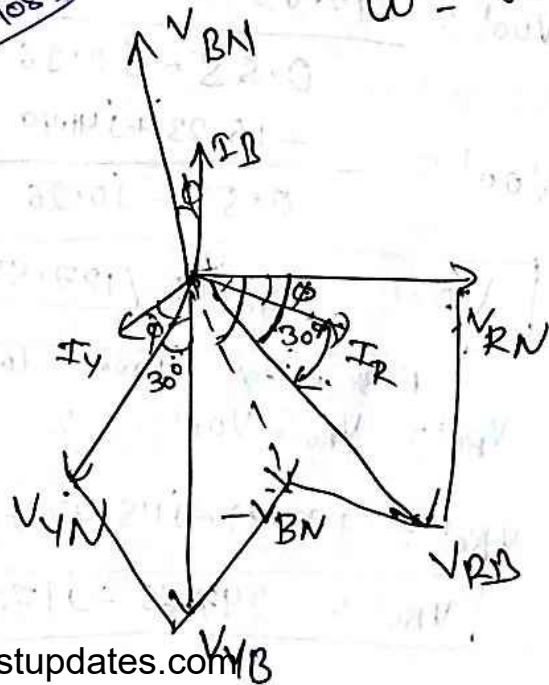
$$I_Y = I_1 - I_2 = 29.26 \angle -6.99^\circ - 23.88 \angle -108.56^\circ$$

$$I_B = -I_2 = 23.88 \angle -108.56^\circ$$

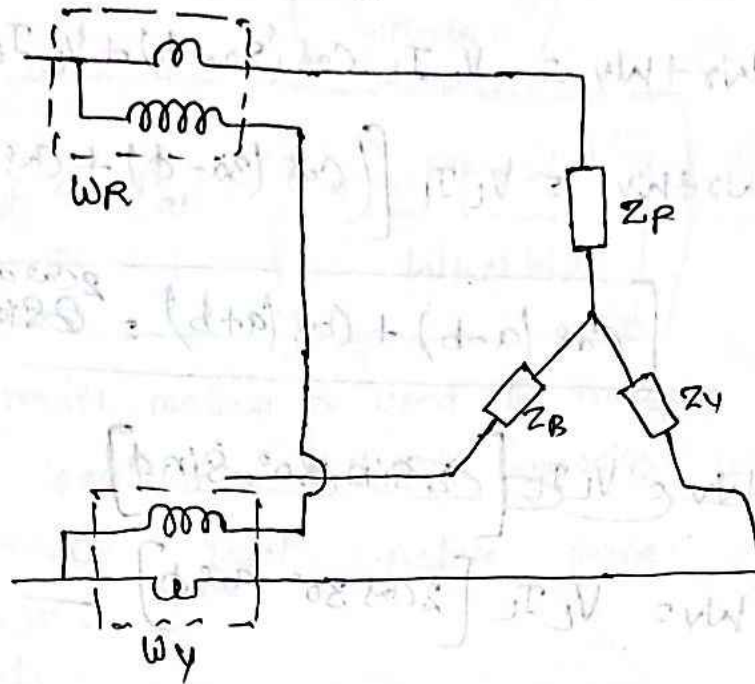
$$V_{RB} = V_{RN} - V_{BN}$$

$$V_{YB} = V_{YN} - V_{BN}$$

$$W = V \cdot I \cos \phi$$



\* Two wattmeter method :-



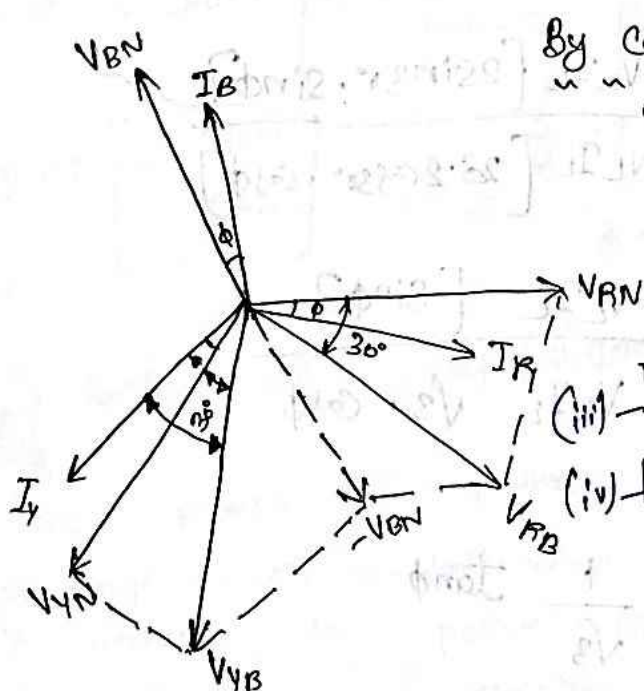
$W_R \rightarrow$  Power reading of wattmeter 1

$W_Y \rightarrow$  Wattmeter 2

Total power =  $W_R + W_Y$

(i)  $W_R = V_{RB} I_R \cos(30 - \phi)$  ( $V_{RB} = V_{RN} - V_{BN}$ )

(ii)  $W_Y = V_{YB} I_Y \cos(30 + \phi)$  ( $V_{YB} = V_{YN} - V_{BN}$ )



By considering

$V_{RB} = V_{YB} = V_L$

$I_R = I_Y = I_L$

Sub them in eqn

(i) & (ii)

(iii)  $W_R = V_L I_L \cos(30 - \phi)$

(iv)  $W_Y = V_L I_L \cos(30 + \phi)$

Equ (iii) + Equ (iv)

$$\Rightarrow W_x + W_y = V_L I_L \cos(30 - \phi) + V_L I_L \cos(30 + \phi)$$

$$W_x + W_y = V_L I_L [\cos(30 - \phi) + \cos(30 + \phi)]$$

$$\cos(a-b) + \cos(a+b) = \frac{2\cos a \cdot \cos b}{\cancel{2\sin a \cdot \sin b}}$$

$$W_x + W_y = V_L I_L [2\sin 30^\circ \sin \phi]$$

$$W_x + W_y = V_L I_L [2\cos 30^\circ \cos \phi] \quad \text{--- (v)}$$

Equ (iii) - Equ (iv)

$$W_x - W_y = V_L I_L \cos(30 - \phi) - V_L I_L \cos(30 + \phi)$$

$$W_x - W_y = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$

$$W_x - W_y = V_L I_L [2\sin 30^\circ \sin \phi] \quad \text{--- (vi)}$$

Equ (vi)  $\div$  Equ (v)

$$\frac{W_x - W_y}{W_x + W_y} = \frac{V_L I_L [2\sin 30^\circ \sin \phi]}{V_L I_L [2\cos 30^\circ \cos \phi]}$$

$$\frac{W_x - W_y}{W_x + W_y} = \frac{V_L I_L [\sin \phi]}{V_L I_L \sqrt{3} \cos \phi}$$

$$\left( \frac{W_x - W_y}{W_x + W_y} \right) = \frac{1}{\sqrt{3}} \tan \phi$$

$$\tan \phi = \sqrt{3} \left( \frac{W_R - W_Y}{W_R + W_Y} \right)$$

$$\phi = \tan^{-1} \left[ \frac{\sqrt{3} (W_R - W_Y)}{W_R + W_Y} \right]$$

A two wattmeter method is used to measure power in a 3-phase load. The wattmeter readings are 400V and -35V. Calculate total active power and power factor.

⇒ Given data

Two wattmeter method is used

$$W_R = 400V, \quad W_Y = -35V$$

$$\text{Total power (P)} = \cancel{W_R} + W_Y$$

(Active power)  $\Rightarrow = 400 - 35 = 365V$

$$\text{Power factor } = \frac{\cos \phi}{\phi} = \tan^{-1} \left[ \frac{\sqrt{3} (W_R - W_Y)}{W_R + W_Y} \right]$$

$$\phi = \tan^{-1} \left[ \frac{\sqrt{3} (400 - (-35))}{400 - 35} \right]$$

$$\phi = \tan^{-1} \left[ \frac{\sqrt{3} (435)}{365} \right]$$

$$\phi = \tan^{-1} (2.06)$$

$$\phi = 64.106$$

$$\text{Power factor } (\cos \phi) = \cos (64.106) = 0.436$$

\* The input power to a three phase load is 10kW at 0.8 power factor. 2-wattmeters are connected to measure the power. Find the individual readings of the wattmeter.



∴ Give data

$$W_r + W_y = 10 \text{ kW} \quad \text{--- (i)}$$

$$\cos \phi = 0.8 \text{ pf}$$

$$\phi = \cos^{-1}(0.8) = 36.86$$

$$\phi = \tan^{-1} \left[ \frac{\sqrt{3} (W_r - W_y)}{(W_r + W_y)} \right]$$

$$\tan(36.86) = \left[ \frac{\sqrt{3} (W_r - W_y)}{10 \times 10^3} \right]$$

$$\tan \times 0.749 = \cancel{0.992} (W_r - W_y) \quad 1.732 \times 10^{-4} (W_r - W_y)$$

$$W_r - W_y = 4328.5 \text{ W}$$

$$W_r - W_y = 4.328 \text{ kW} \quad \text{--- (ii)}$$

from eqn (i) & (ii)

$$W_r + W_y = 10 \text{ kW}$$

$$W_r - W_y = 4.328 \text{ kW}$$

$$\hline 2W_r = 14.328 \text{ kW}$$

$$W_r = 7.164 \text{ kW}$$

$$7.164 + W_y = 10$$

$$W_y = 10 - 7.164$$

$$W_y = 2.836 \text{ kW}$$

\* Two wattmeters are connected to measure the power in 3-phase circuit the reading of each of the meter is 5 kW when the load power factor is unity. If the power factor at the load is change to 0.707 lagging

Without changing the total input power calculate the readings of the two wattmeter method.

⇒ Given each wattmeter reading is 5 kW

So,  $w_x + w_y = 10 \text{ kW}$   $w_x + w_y = 10 \text{ --- (i)}$

Cal  $\phi = 0.707$

At unity power factor  $= w_1 + w_2$

$\phi = \cos^{-1}(0.707) = 45.008$

$\phi = \tan^{-1} \left[ \sqrt{3} \frac{(w_x - w_y)}{(w_x + w_y)} \right]$

$\tan(45.008) = \left[ \frac{\sqrt{3} (w_x - w_y)}{10} \right]$

$\frac{1.0}{0.173} = w_x - w_y$

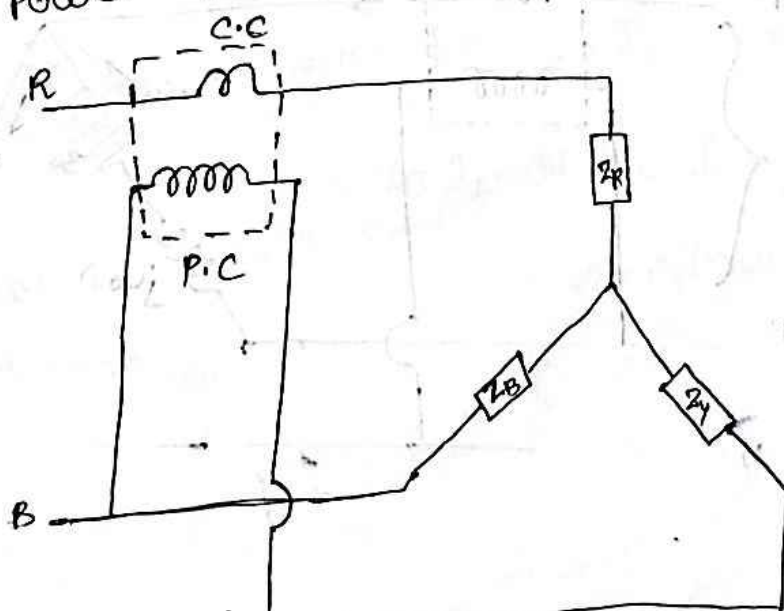
$5.780 = w_x - w_y \text{ --- (ii)}$

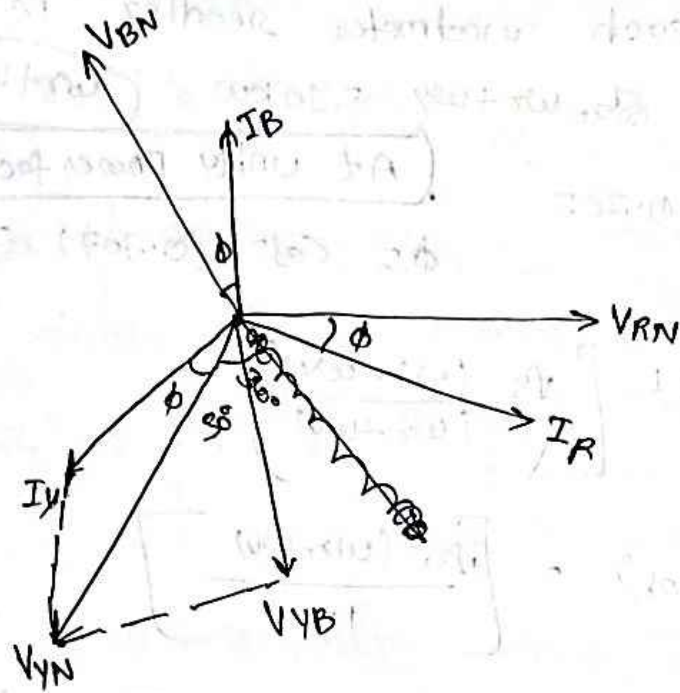
$w_x + w_y = 10$        $w_x = 7.890 \text{ kW}$

$w_x - w_y = 5.780$        $w_y = 10 - 7.890$

$2w_x = 15.780$        $w_y = 2.11 \text{ kW}$

\* Reactive Power measurement in 3-phase circuit.





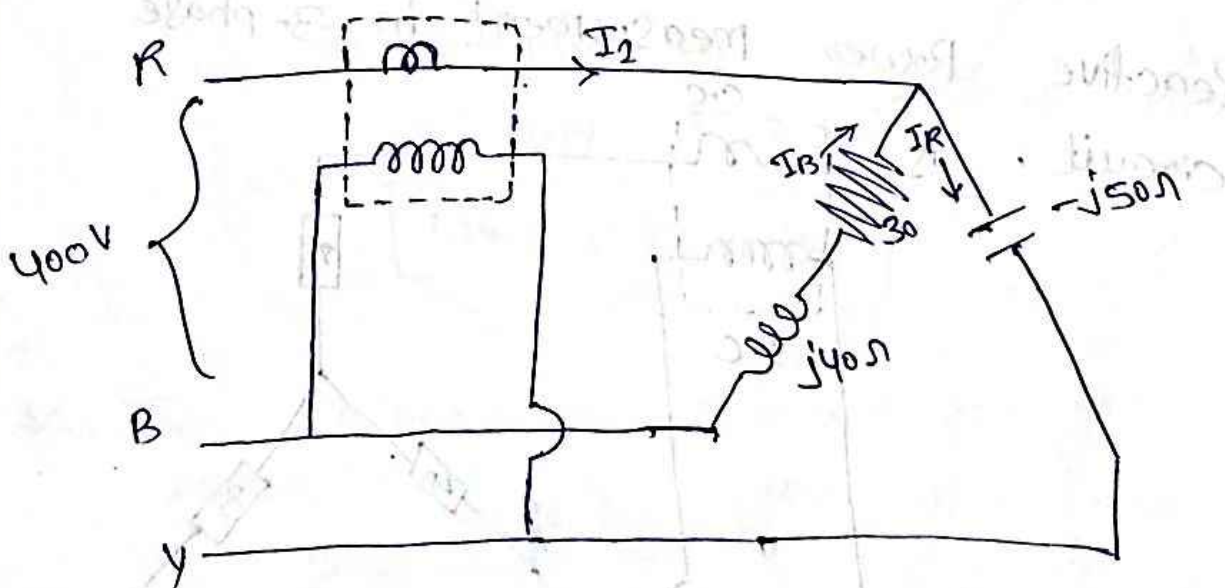
$$V_{YB} = V_{YN} - V_{BN}$$

$$\text{Wattmeter reading}_1 = V_{YB} \cdot I_R \cos(90^\circ - \phi)$$

$$V_{YB} = V_L$$

$$I_R = I_L$$

$$\text{Wattmeter reading} = V_L I_L \sin \phi$$



Determine the <sup>reading of the wattmeter.</sup> ~~reactive~~ power in the circuit which is connected to 400V 3-phase supply and also draw the negative phasor diagram.

∴ for from fig.  $I_1 + I_B = I_R$

$$I_1 = I_R - I_B$$

$$V_R = 400 \angle 0^\circ$$

$$V_Y = 400 \angle -120^\circ$$

$$V_B = 400 \angle -240^\circ$$

$$I_R = \frac{V_B}{Z_R} =$$

$$I_R = \frac{400 \angle 0^\circ}{0 - j50} = \frac{400 \angle 0^\circ}{50 \angle -90^\circ} = 8 \angle 90^\circ \text{ A}$$

$$I_B = \frac{400 \angle -240^\circ}{30 + j40} = \frac{400 \angle -240^\circ}{50 \angle 53.13^\circ} = 8 \angle -293.13^\circ \text{ A}$$

$$I_1 = I_R - I_B$$

$$= 0 + j8 - 3.14 - j7.35$$

$$= -3.14 + 0.65j$$

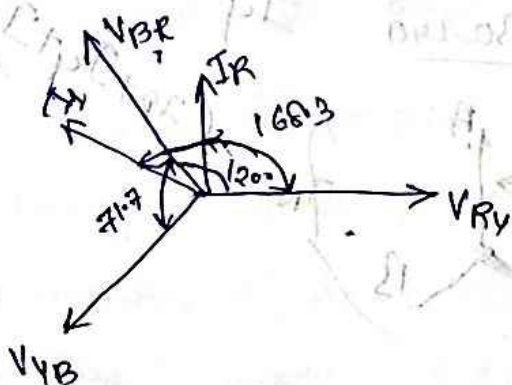
$$I_1 = 3.20 \angle 168.30^\circ \text{ A}$$

Reactive power measurement =  $V_{YB} I_1 \cos(\angle V_{YB} - I_1)$

$$= V_{YB} I_1 \cdot \cos(\angle V_{YB} - I_1)$$

$$= 400 \times 3.2 \times \cos(71.7^\circ)$$

$$= 401.91 \text{ W}$$

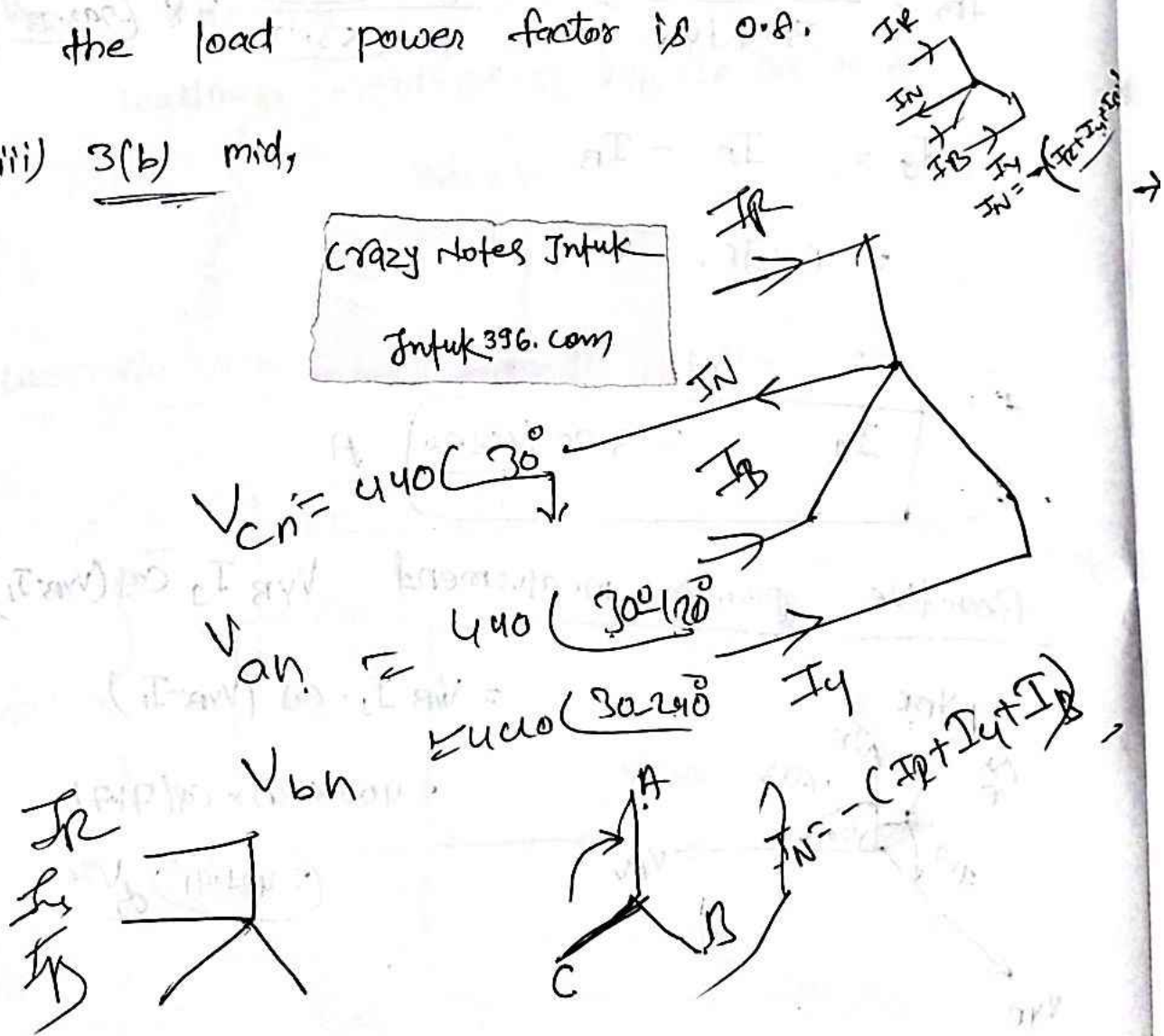


# Unit - II ASSIGNMENT

- (i) 3 impedance  $(7+j4)\Omega$ ,  $(3+j2)\Omega$  &  $(9+j2)\Omega$  Connected in between neutral and the ~~red~~ yellow and blue phase respect of 3-phase 4-wire system the line voltage is 440V  
 Calculate (i) The current in line  
 (ii) The current in neutral wire.

- (ii) Calculate the total power input and reading of the two wattmeter connected to measure power in 3-phase balanced load. If the reactive power input is 15 KVAR and the load power factor is 0.8.

(iii) 3(b) mid



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